Forum Acusticum 2011 - Aalborg - Denmark

A Comparison of Saxophone Impedances and their Playing Behaviour

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Abstract
Input impedances (and reflectances) of woodwind-like instruments are measured with a system of six microphones, calibrated with three non-resonant loads, and a least-mean-square signal processing technique. The high accuracy of this measurement technique enables the detection of subtle differences between objects. The measurements are compared to theoretical impedances calculated using the Transmission Matrix Method (TMM), from which we discuss the accuracy of the TMM and point out some discrepancies. From the features of the impedance or reflectance, various properties of woodwind instruments may be obtained, such as the resonance frequencies, the tonehole cutoff frequency, the effect of a register hole and the harmonicity of a resonance. These features are compared for a few alto saxophones with fairly distinct playing behaviours, shedding some light of the relationship between the geometry of an instrument, its input impedance and the quality of the instrument.

PACS no. 43.75.Ef,43.75.Yy

1. Introduction

The objective of this research is to better understand the relation between the geometry of alto saxophones, their input impedance and their playing behavior. The most important aspects of this study are the tuning of the instrument, its "response" and its overall tone quality.

In this paper the specified note names correspond to the written pitch of an alto saxophone, for which a written C sounds E♭ in concert pitch. The lowest note is B0♭ with a frequency of 138.6 Hz. The highest note of the first register is D2♭ with a frequency of 329.6 Hz and the highest note of the second register is F3 or G3♭ with a frequency of 830.61 Hz or 880.00 Hz. The tonehole for the G3♭ does not exists on older instruments.

An alto saxophone is composed of three parts, the mouthpiece, the neck (or crook) on which a register hole is located, and the body, which is terminated by a bell. The bore of the instrument is approximately conical in shape with a number of small deviations, including two bends, one on the neck and another just before the bell of the instrument. The instruments are equiped with 22 or 23 toneholes, three of which are used for alternate fingerings, and two register holes. The farthest three toneholes, located before the bell and just after the bent portion of the body (also called the elbow), are used to play the lowest four notes of the instrument (B0♭, B0, C1 and D1♭) and they are not normally used in the second register. The following twelve toneholes (excluding the 3 toneholes used for alternate fingerings) produce the next twelve notes by semitone steps (D1 to D2♭). These same twelve toneholes also produce the next twelve notes, one octave higher, with the aid of the register holes. Four or five other toneholes are used to support the notes D3 to F3 or G3♭. These are only used in their second register with the second register hole. They are not designed to be used in their first register. The first register hole, located near the top of the body, is used for the notes D2 to A2♭ and the second register hole, located on the neck, is used for the notes A2 to F3 or G3♭. There is only one key to activate both register holes, with a mechanical system that automatically opens the proper register hole based on the fingering.

In general, an open tonehole is followed by a series of more open toneholes. For some notes, the first open tonehole is followed by one or two closed toneholes before a row of more open toneholes. This may change the acoustic behavior of the instrument and usually requires this first open tonehole to be larger in diameter. There are five fingerings where this occurs. For D1, E1, G1♭ and A1, there is one closed tonehole after the first open tonehole. For C2, there are two closed toneholes after the first open tonehole. These same fingerings are used in the second register as well.

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The playing frequencies of an instrument correlate well with the resonance frequencies of the air column corrected for the effect of the excitation mechanism. The "response", or ease of play, generally correlates with the magnitude of the resonances and with a proper alignment of the resonances (Gazengel, 1994; Dalmont, Gazengel, Gilbert, & Kergomard, 1995). This is particularly important for the lowest notes of the instrument, for which the air column supports many resonance frequencies. When the magnitude of the second resonance is of the same magnitude or larger than that of the fundamental, inharmonicity makes the note difficult to play and/or produces a multiphonic. If the second resonance is smaller in magnitude, it mainly affects the tone colour. For the notes in the second register, the higher resonances are weak because they are near or above the cutoff frequency of the tonehole lattice, therefore, the proper alignment of the higher resonances is of little importance in this case.

To better understand the relation between an instrument's acoustic properties, quality and geometry, we make use of measurements and Transmission-Matrix Method (TMM) calculations of its input impedance. We measured the input impedance for each fingering of three alto saxophones with distinct playing bahaviors and different geometries: a Selmer Model 26 (M26) (serial 8821, made in 1928), a Selmer Super Series (SS) (serial 12249, made in 1930) and a Selmer Super Action Series II (SAS2) (serial 438024, made in 1989). From these measurements, the instruments are evaluated using the criteria mentioned above. This is compared with the results of TMM calculations of the geometry of the SAS2, as measured by the authors, and with a subjective evaluation of the playing behavior of the instruments.

Using physical measurements as well as a bore reconstruction algorithm (Sharp, 1996), we identified some important differences in the geometry of these instruments. First, the SS has a significantly larger bore than the other two instruments. The internal diameter of the neck at its larger end is 0.5 mm wider than for the other two instruments. The rest of the instrument is larger by the same amount up to the bell. The M26 and the SAS2 have very similar geometries but the neck volume of the M26 is larger, this extra volume being located near the bend in the neck.

From the impedance measurements, we can compare a number of features: the resonance frequencies, the magnitude of the resonances and the harmonicity of the resonances. The resonance frequencies are obtained from the zeros of the imaginary part of the reflectance, using linear interpolation between the measurement data points. The magnitude of the impedance at resonance is also calculated from the interpolated reflectance, converted to an impedance with $Z/Z_0 = (1+R)/(1-R)$, where $Z_0 = \rho c/S$. The harmonicity is calculated as the frequency deviation in cents of the second resonance relative to twice the fundamental resonance frequency.

2. Measurement System and Method

The impedances were measured with a multi-microphone measurement system based on a least-mean-square signal processing technique. The impedance tube is excited with a JBL 2426 horn driver. Six PCB Piezotronics condenser microphones (model 377B10) with preamplifiers (model 426B03) are flush mouted with the inner wall of the tube at 30 mm, 60 mm, 100 mm, 150 mm, 210 mm and 330 mm from the input plane of the object. The microphones are connected with a PCB Piezotronic signal conditioner (model 483C30) and then to the computer through a RME FireFace 800 audio interface. The signals are sampled at 48 kHz. The system is excited with a looping logarithmic swept sine tone. The length of one period of this looping sound is equal to the length of the Fourier transform. The responses to each repetition of the swept sine are averaged together in the time domain before any processing is performed and the first response is dropped. The spectral analysis uses 38768 points, giving a frequency resolution of 1.46 Hz If any harmonic distortion was present in the system, it is removed using the method presented by Farina (2000).

Using the pressure spectra at each microphone, an algorithm making use of the Moore-Penrose Pseudo Inverse is solved for the forward and backward traveling waves (Jang & Ih, 1998), thus effectively measuring the reflectance of the object, which is easily converted to an impedance if desired.

The apparatus is calibrated with three non-resonant loads with the procedure described by Dickens, Smith, and Wolfe (2007) but the pipe of infinite length is replaced by a 2 meter long pipe and a procedure to time-window the first impulse from the reflections, thus simulating an infinitely long pipe, as described by Kemp, Walstijn, Campbell, Chick, and Smith (2010).

The input impedances were measured at the input of the saxophone neck. This ensures that we measure the properties of the saxophones without the perturbations of the mouthpiece, which may vary significantly in shape for different models. A cylindrical segment is artificially added to each measurements to simulate the presence of a mouthpiece.

3. Results and Discussion

From the measured or calculated input impedances, indicators of the intonation, ease of play and tone colour are proposed and discussed in relation with informal playing experiments. The results of the TMM

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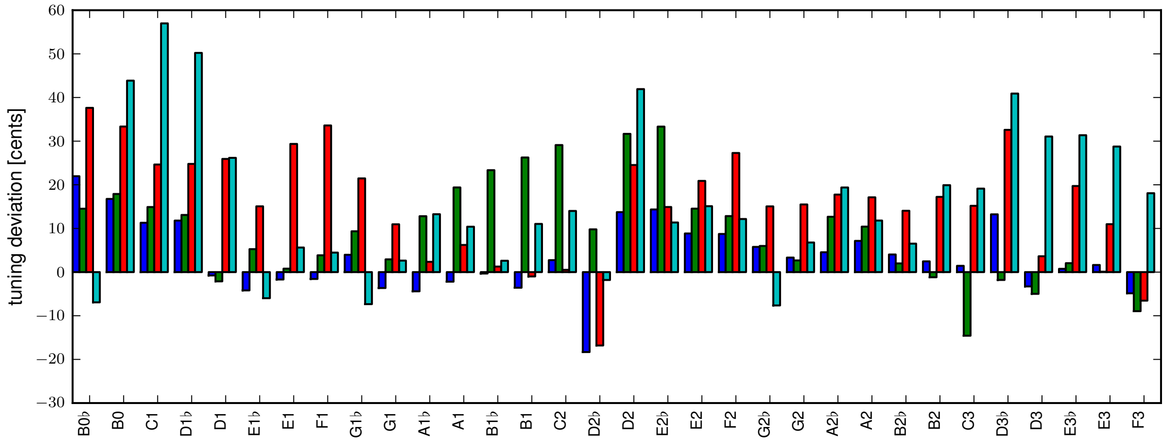

Figure 1. Frequency deviation in cents of the resonance frequencies supporting each note relative to their equal-tempered frequencies. Measurements: M26 in red, SS in green and SAS2 in blue. TMM of SAS2 in cyan.

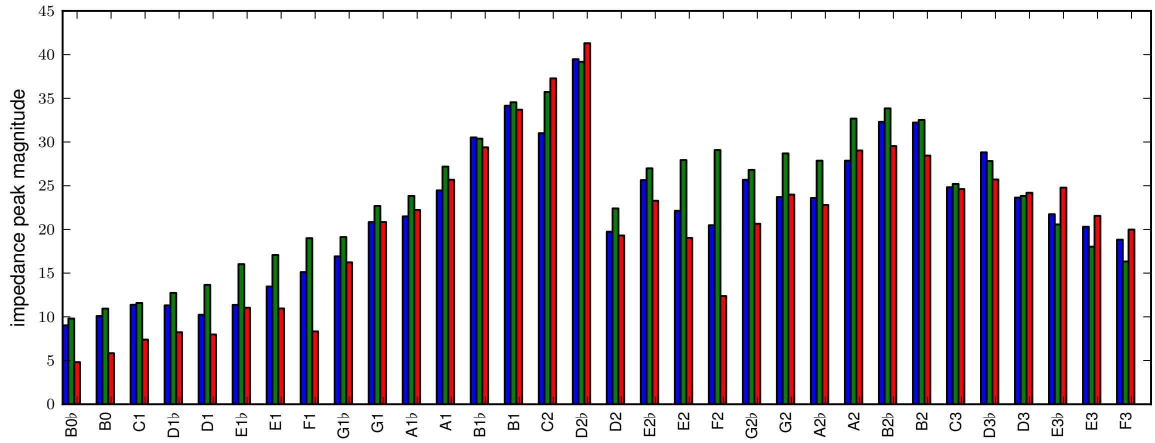

Figure 2. Comparison of the magnitude of the fundamental impedance resonance for the three measured saxophones: M26 in red, SS in green and SAS2 in blue.

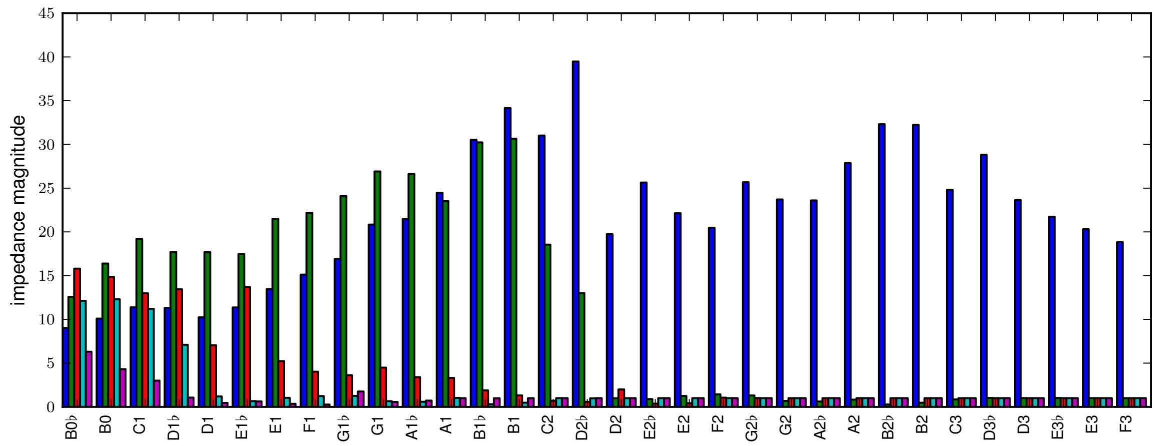

Figure 3. Magnitudes of the first five impedance resonances of the Selmer Super Action Series II saxophone

calculations are discussed after the analysis of the measurements.

3.1. Intonation

The intonation of the instruments (playing frequencies) were estimated from the resonance frequencies of the air column. For each instrument, the mouthpiece and excitation mechanism are simulated by a straight cylinder, the length of which was adjusted differently for each instrument in order to minimize the pitch deviations in the extremes of the instrument, as was done by (Nederveen, 1969/1998). The lengths are

62 mm for the SAS2, 67 mm for the SS and 70 mm for the M26.

Figure 1 displays the frequency deviations for each of the three measured instruments and the TMM calculations. The fundamental resonances for the lowest four notes of all three instruments are sharp, with the M26 being the worst. For the next twelve notes, the most modern instrument (SAS2) provides accurate resonances within ±5 cents, apart for the D2♭, which is 20 cents flat. Some resonances of the other two instruments are sharp by up to 30 cents, though the others are relatively well tuned. The D2♭ resonance is flat on all instruments except the SS.

The second register of the instruments starts on the D2 and is played with register keys. Once again, the SAS2 presents the smallest deviations of all three instruments, even though the first 4 notes (D2 to F2) are sharp by about 10 cents (as well as the D3♭). In the second register, the M26 has resonances that are sharp by 15 to 25 cents for all notes. The SS has very sharp resonances (+25 cents) on D2 and E2♭, and is flat (-20 cents) on C3.

Results of our informal playing experiments generally agree with these observations: the lowest 4 notes of all instruments are indeed sharp, the D2♭ is flat, the first 4 notes of the second register are sharp relative to the first register and D3♭ is sharp.

3.2. Response

The first indicator that we propose to use for the "response", or "ease of play", of the instruments is the magnitude of the impedance at the fundamental resonance frequency of each note. Figure 2 compares these magnitudes for the three saxophones. For the lowest 8 notes, the M26 presents significantly smaller resonance magnitudes, suggesting that this instrument should be more difficult to play. For notes D1 to A1, the SAS2 also has weaker resonances, suggesting that it should also be somewhat more difficult to play (but not as much as the M26). According to this analysis, the SS should be the easiest instrument to play. In the second register, the M26 has significantly weaker resonance of all saxophones. The SAS2 has lower impedance magnitudes from D2 to A2. Again, the SS should respond better.

Figure 3 displays impedance magnitudes of the first five resonances for the SAS2. For the first four notes, the air column provides support up to the fourth harmonic of the fundamental. For the next twelve notes of the first register, the fundamental and the first harmonic are well supported by the air-column and there is a significant amount of third resonance for D1 to A1. Then, for the second register, only the fundamental is supported by a resonance of the air-column. For all notes below A1, the second resonance is stronger than the fundamental resonance, which is typical of the saxophone and often causes the playing frequency to jump to a higher partial. A proper embouchure setting is of primary importance to play the lower notes of the instrument.

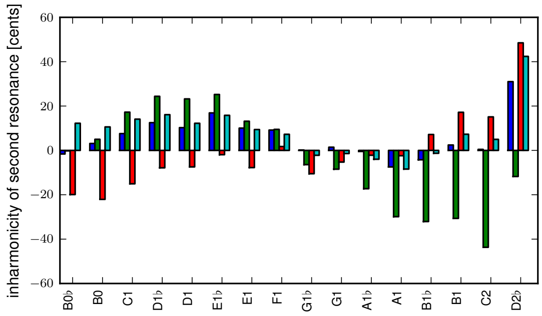

Figure 4. Frequency deviation in cents of the second resonance frequencies for each note of the first register of the saxophones. Measurements: M26 in red, SS in green and SAS2 in blue. TMM of SAS2 in cyan.

Another indicator of the "response" is the harmonicity of the resonances of the air column. This is particularly important for the notes in the first register, because upper resonances are strong; if they are mis-aligned, it renders the notes unstable, with a tendency to produce a multiphonic tone. Furthermore, the notes of the second register are based on the second resonance of notes in the first register, therefore, the proper tuning of the second register is tightly related to the harmonicity of the first resonance. The harmonicity for all notes of the first register is displayed in Fig. 4. All instruments present deviations from perfect harmonicity for some of their notes. We observe that the SS has the strongest resonances of all three instruments but quite a bit of inharmonicity, and this likely reduces the quality of its response. For the second register, the harmonicity of the resonances is of little importance, because these resonances are weak (see Fig. 3).

The playing experiments confirm that the M26 is more difficult to play. The lowest note of the instrument has a tendency to produce multiphonic tones and only with embouchure manipulation we can produce the normal notes. The SS and SAS2 are quite similar in terms of response.

3.3. Tone colour (timbre)

There are subtle variations in tone colour among the instruments and among each note of an instrument. The possible causes of such variations are many: geometry of each individual tonehole, height of the keys, slight inharmonicity of the resonances due to the shape of the bore, extra damping added by small leaks, etc. Each of these factors would produce changes in the input impedance curves of the instruments. Therefore, it is desirable to be able to compare the tone colour of two fingerings of a single instrument

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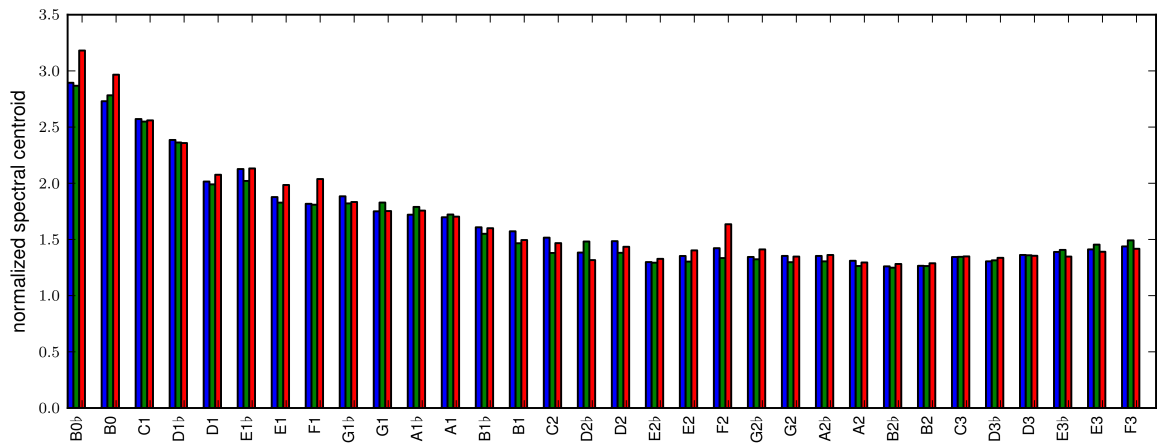

Figure 5. Normalized spectral centroids

or different instruments. Such a criteria would be useful while designing an instrument, in order to have an instrument with an even timbre and to control the overall brightness of the tone.

As a potential method to compare the tone colour of two instruments, we investigated a spectral centroid calculation performed on the input impedance as:

$$\text{centroid} = \frac{1}{f_0} \frac{\sum A_k f_k}{\sum A_k}, \qquad (1)$$

where f_k is the frequency of each peak ($f_k = kf_0$) based on multiples of the fundamental resonance frequencies and where A_k is the amplitude of the resonance at f_k. The result for each note of the three saxophones is displayed in Fig. 5. The differences are quite subtle, despite a fairly strong variation of timbre when the instruments are played. It is not clear that an adequate estimate of timbre can be calculated from the input impedance, since this measure refers to the internal pressure spectrum rather than the radiated spectrum. That is, the input impedance contains very little energy above the tonehole lattice cutoff frequency and our perception of timbre is likely strongly influenced by frequencies above this limit. Further consideration is necessary on this topic.

3.4. Measured vs. Calculated Impedances

Figures 6 to 9 compare input impedances measured using the approach described in Sec. 2 with those computed using the TMM for the SAS2 geometry. Figure 6 displays the results when all toneholes are closed. The number of resonances supported by the air column is large. For this fingering, it is important that the resonances are well aligned. In the case of the M26, the fundamental resonance frequency is sharp but all upper resonances are well aligned, which may explain the seeming difficulty in producing this note. The TMM calculation for this geometry produces reasonable agreement in terms of impedance peak frequencies, but it appears to significantly underestimate the damping in the system.

Figure 7 corresponds to the note C2. This fingering has one open tonehole followed by two closed toneholes and as a consequence, the third resonance is very weak. This is not the case for the D2♭, one semitone above, displayed in Fig. 8. The second resonance is well aligned for C2 and 25 cents sharp for D2♭ (see Fig. 4). These two factors may explain the differences in response of these two notes.

Figure 9 corresponds to the note A2, which is based on the second resonance of A1. The TMM calculation does not accurately predict the displacement of the fundamental resonance caused by the register hole, most likely because the standard tonehole model is inadequate when used to model a register hole.

In general, the TMM predicts stronger resonances for all fingerings compared to the measurements. This is particularly obvious near and above 1 kHz where resonances are often mostly absent from the measurement but are still strong in the calculation (see Fig 7 for instance). This is possibly a consequence of tonehole interactions, which are neglected in the TMM.

It should be noted that the results of the TMM calculations are very sensitive to the description of the geometry, particularly the diameter of the bore as a function of the distance along its spine. Errors as small as 0.05 mm may change significantly the resonance frequencies, particularly in the neck of the instrument.

The TMM calculations were based on the SAS2 geometry but the results differ significantly from the corresponding measurements. It is possible that the geometry was not measured with a sufficient accuracy due to the difficulty in obtaining the dimensions of the bore from measurements made on the outside of the instrument. It is also likely that the theory does not always provide correct results, such as for register holes or in bent portions of the instrument.

4. Conclusion

This paper presents preliminary results toward a better understanding of the relations between the input

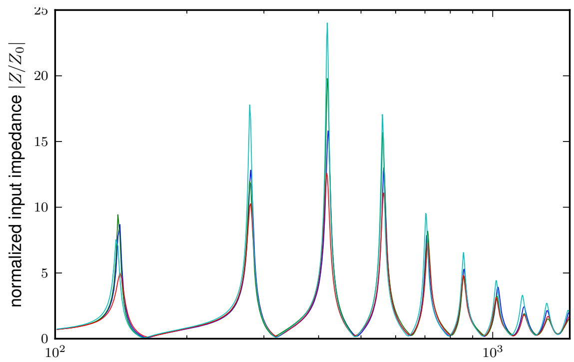

Figure 6. Impedance of B0♭

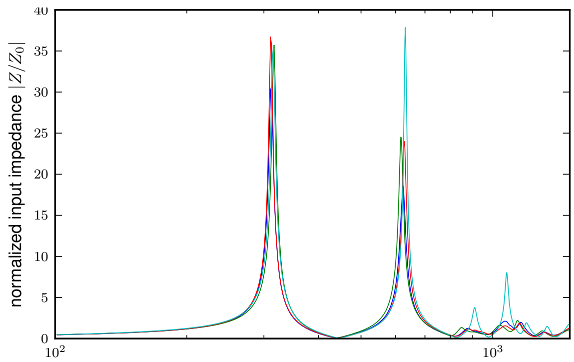

Figure 7. Impedance of C2

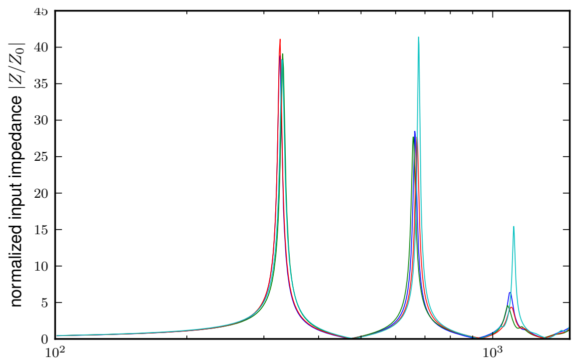

Figure 8. Impedance of D2♭

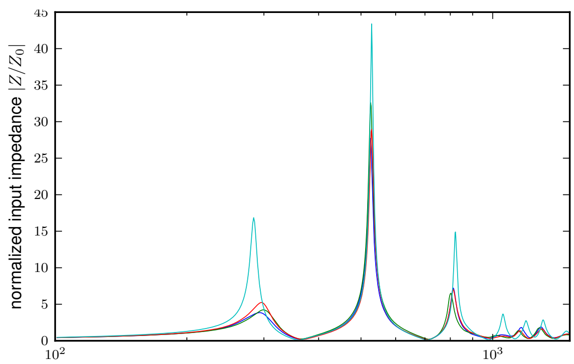

Figure 9. Impedance of A2

impedance of alto saxophones, their playing behaviors and their geometries. More work is required in calculating the properties of an instrument using the TMM because the discrepancies between the measurement and the calculations are still significant.

A rigorous playing experiment with a larger number of subjects is required for an evaluation of the playing frequencies, ease of play and timber of the instruments.

Furthermore, due to the difficulty in measuring accurately the dimensions of existing instruments, we are planning to fabricate the instruments using a molding process that would ensure great dimensional accuracy of the prototypes.

Acknowledgement

This project was funded in part by a Discovery Accelerator Supplements grant from the Natural Sciences and Engineering Research Council of Canada. Thanks to Mathieu Leclair of the Quasar Saxophone Quartet for providing informal playing evaluations.

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