COUPLING LATTICE BOLTZMANN MODELS TO DIGITAL WAVEGUIDES FOR WIND INSTRUMENT SIMULATIONS

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Abstract

The lattice Boltzmann method (LBM) is a valuable numerical tool in musical acoustics, particularly when modeling wind instruments for which the interaction between the flow and the acoustic field is important. This paper describes an approach for addressing two limitations of LBM, namely computational cost and the inability to directly impose acoustical boundary conditions at open boundaries. The technique consists in simplifying the system by representing the parts where complex fluid-acoustic interaction takes place with LBM, whereas regions that are well approximated by linear wave propagation are represented with a digital waveguide (DWG). The example provided consists of a clarinet-like system whose mouthpiece is represented by a LBM model coupled to a DWG, which represents the instrument's bore. The junction implementation between the LBM model and the waveguide, as well as the conversion of physical variables into wave variables, is detailed.

INTRODUCTION

The behavior of wind instruments is strongly dependent on the interaction between the flow and the acoustic field. These interactions explain several non-linear phenomena such as self-sustained oscillations in jet instruments, edge tones and nonlinear dissipation due to vortex shedding at the instrument discontinuities.

The lattice Boltzmann method (LBM) is a useful tool to represent these systems due to its ability to resolve in a single time step the different scales associated with the flow and the acoustic field, thus facilitating the representation of the nonlinear phenomena previously mentioned.

A significant number of LBM studies in music acoustics have been conducted beginning with the pioneering work of Skordos (1995), who represented the interaction between the fluid flow and the acoustic field in organ pipes. Buick et al. (1998, 2000) simulated the propagation of linear sound waves and later, simulated the formation of shock waves using different boundary condition schemes. Kuehnelt (2003) investigated the mechanisms of sound production in the flute using a three-dimensional LBM model. Atig (2004) represented the vortex shedding at duct terminations and Neal (2002) simulated flow aspects in lip-mouthpiece systems of brass instruments. Da Silva and Scavone (2007) proposed an axisymmetric LBM model to predict the acoustic radiation of ducts. More recently, da Silva et al. (2007) derived a single-reed mouthpiece model involving a moving boundary based on a distributed model of the reed.

One important drawback of LBM in acoustics is the inability to specify an arbitrary boundary condition at a discontinuity. This problem is encountered when representing the radiation impedances at bore apertures, such as toneholes and open ends. An accurate but rather expensive solution for this problem involves including in the LBM model the radiation domain around the open end (da Silva and Scavone, 2007).

This paper presents a technique to simplify the representation of radiation impedances at open boundaries by connecting the LBM grid to a low-order digital filter through a digital waveguide (DWG). As an example, we use a clarinet-like system whose mouthpiece is represented by LBM and connected to a waveguide representing the bore. The radiation impedance in the end of the waveguide is approximated by a low order digital filter, as proposed by Scavone (1999).

OVERVIEW OF THE NUMERICAL TECHNIQUES

Lattice Boltzmann Method (LBM)

The lattice Boltzmann (LB) is classified as a *nonequilibrium* method whereby the fluid domain is investigated at a particle level. It was derived from the cellular automata method by implementing a simplification of the Boltzmann equation to describe simple collision rules to conserve mass and momentum. A full description of the lattice Boltzman theory can be found in Wolf-Gladrow (2000) and Succi (2001).

In this paper we use the D2Q9 model, after Qian et al. (1992). This model is represented by a two-dimensional squared lattice with 9 sites. Each site connects to a neighbor lattice by a unity vector c_i, where i = 1, 2, ..., 8, indicates the site number, with the exception of the rest site i = 0, represented by the null velocity vector c_0. The discrete Bolzmann equation uses the simplified collision function known as LBGK defined with a single relaxation time τ, and given by

$$f_i(x + c_i, n + 1) - f_i(x, n) = -\frac{1}{\tau}(f_i - f_i^M) \tag{1}$$

where f_i are distribution functions associated with the vectors c_i at the site x and discrete-time n. f_i^M are the equilibrium distribution functions which depend on the local fluid velocity $u(x, n)$ and local fluid density $\rho(x, n)$. The general expressions of the equilibrium functions f_i^M associated with the D2Q9 model are

$$f_i^M = \begin{cases} \rho w_i \left[1 + 3c_i \cdot u + \frac{9}{2}(c_i \cdot u)^2 - \frac{3}{2}u^2 \right] & \text{for } i = 1,2,\ldots,8 \\ \rho \left[\frac{4}{9} - \frac{2}{3}u^2 \right] & \text{for } i = 0 \end{cases} \tag{2}$$

with $w_{1,\ldots,4}$ = 1/9 and $w_{5,\ldots,8}$ = 1/36.

The local macroscopic variables ρ and u are obtained in terms of moments of the local distribution function f_i by

$$\rho(x,n) = \sum_i f_i(x,n) \qquad \rho(x,n)u(x,n) = \sum_i f_i(x,n)c_i . \tag{3}$$

Other macroscopic variables such as pressure p, viscosity, ν and speed of sound c_s are obtained when recovering the Navier-Stokes and continuity equations from Eq. (1) via a Chapman-Enskog expansion and, in the case of the D2Q9 model, are given by

$$p = (\rho - \rho_0)c_s^2, \qquad \nu = \frac{2\tau - 1}{6} \qquad \text{and} \qquad c_s = \frac{1}{\sqrt{3}} . \tag{4}$$

Digital Waveguide Technique and Digital Filters

Digital waveguide techniques have been well documented for applications in musical acoustics (Smith, 1992). Their essential feature is the use of digital delay lines to simulate lossless traveling-wave propagation. In a one-dimensional context, as applied here, they are especially efficient because only one or two digital delay-lines are necessary to model an air column, as illustrated in Fig. 1 below. The digital filter $R_L(z)$ implements the boundary condition at the end of the air column and a good continuous-to-discrete time fit typically requires only a first- or second-order system. Losses can be incorporated and implemented at discrete locations in the model. For example, it is common to combine propagation losses for travel along one length of the air column with $R_L(z)$.

For the boundary condition represented by the open end of a cylindrical pipe, the results of Levine and Schwinger (1948) have been evaluated and represented in terms of a frequency-dependent reflectance that is used to design a discrete-time digital filter, as described in Scavone, 1999.

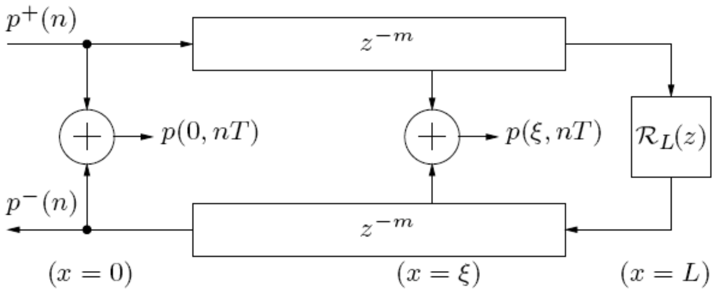

Figure 1: A one-dimensional digital waveguide model of a cylindrical air column.

CONNECTING LBM AND DWG

The Absorbing Boundary Condition

Although we have opted to use the D2Q9 lattice Boltzmann model, the technique presented here can be extended to other LMB schemes without loss of generality. The connection between a two-dimensional LBM model and the one-dimensional DWG involves assuming that the outgoing and incoming components of the wave at the junction are planar and that the flow is one-dimensional. This assumption can be reinforced by placing the connection point far enough from geometrical discontinuities so as to allow the eventual vortical behavior of the flow to decay as it approaches the junction.

The core of this implementation is based on the use of a fairly well known technique in computational fluid dynamics called absorbing boundary condition (ABC). This technique has been adapted to LBM by Kam et al. (2006) and consists of a buffer placed between the lattice domain and the open boundary to create an asymptotic transition towards a prescribed target flow (see Fig. 2). Consequently, the outgoing waves are completely absorbed as they move into the buffer, whilst incoming waves can be prescribed in terms of target distribution functions f_i^T.

The ABC buffer is implemented in the lattice domain by adding an extra term to Eq. (1), resulting in

$$f_i(x + c_i, n + 1) - f_i(x, n) = -\frac{1}{\tau}(f_i - f_i^M) - \sigma(f_i^M - f_i^T), \tag{5}$$

where $\sigma = \sigma_m(\delta / D)^2$ is the absorption coefficient, σ_m is a constant, normally 0.3, δ is the distance measured from the beginning of the buffer zone and D is the width of the buffer. f_i^T is obtained in a similar way as f_i^M, using Eq. (2). In this case, the local velocity u and local density ρ are replaced by the desired target velocity u^T and target density ρ^T, respectively.

Implementing the Junction between the Waveguide and LBM Models

The implementation of the junction demands two simultaneous operations at every time step, namely the determination of the outgoing wave component in the lattice domain immediately before the ABC buffer and the prescription of the incoming wave component in terms of target values at ABC.

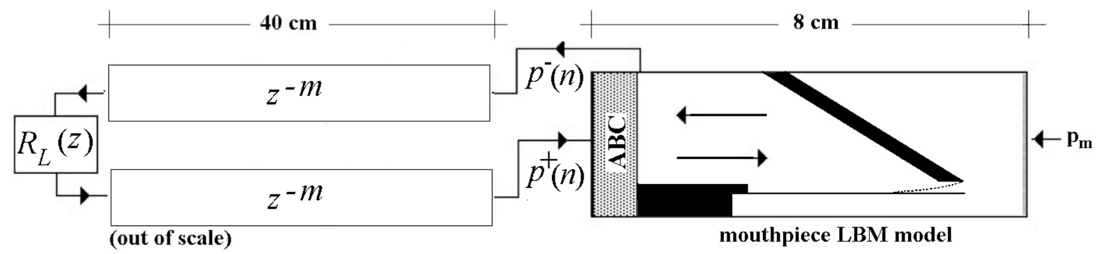

Figure 2: Scheme of the junction between a LBM grid and the DWG.

Assuming plane waves at the junction, the outgoing wave component is obtained from the LBM domain by the solution of the plane wave equation, expressed in terms of pressure by

$$p^- = \frac{1}{2}\left[(\rho - \rho_0)c_s^2 - u_x \rho_0 c_s\right] \tag{6}$$

where ρ_0 is the density of the undisturbed fluid. The density ρ and the horizontal component of the particle velocity u_x are obtained at any vertical point of the lattice, promptly before the ABC buffer (see Fig. 2). The outgoing pressure component p^- propagates along the waveguide, gets reflected according to the boundary condition represented in the digital filter $R_L(z)$ and arrives at the ABC buffer as the incoming pressure component p^+. This value is used to determine the target functions f_i^T at the ABC boundary by assuming $\rho^T = p^+/c_s^2 + \rho_0$ and $u^T = \rho c/\rho_0$.

An important detail should be taken into account when determining the number of elements m in the segment represented by the waveguide. In the lattice domain, the sound wave propagates a distance equal to Δ_x/c_s per time step, whereas in a simple DWG the wave propagates Δ_x for the same time interval, where Δ_x is the space discretization in the lattice domain. Thus, the right number of elements m in a DWG segment of length L is given by $m = L/\Delta_x c_s$.

RESULTS

Impulse Response in a Closed-Closed Pipe

In this section we compare the impulse responses of two models of a closed-closed pipe with length L = 11 cm and radius r = 3 cm. The first model consists of a two-dimensional LBM segment with length L_{LBM} = 6 cm connected to a digital waveguide of L_{DWG} = 5 cm using the technique previously presented (Fig. 3-a). The relaxation time τ in the LBM model was chosen to produce a kinematic viscosity close to that of air at 30 C°.

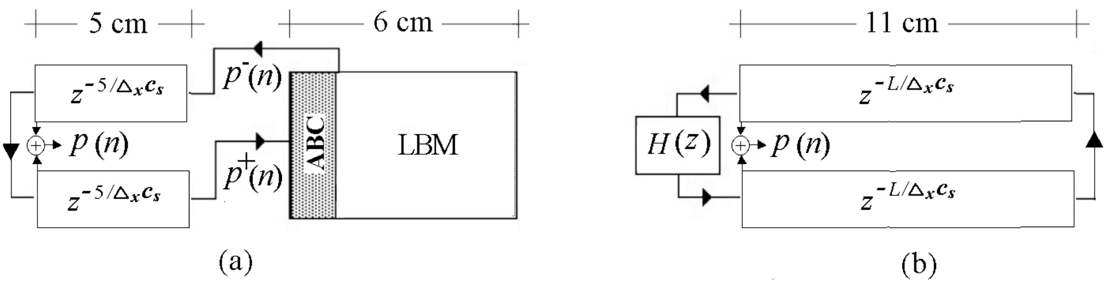

Figure 3: Different representations of a closed-closed pipe model.

The second model is represented by a simple waveguide connected to a digital filter $H(z)$ as proposed by Scavone (1999) (Fig. 3-b). The role of the filter is to create the same viscous dissipation intrinsic to the LBM segment of the first model, so that their impulse responses can be compared. Both models are initiated with a Gaussian impulse whose amplitude is measured at the very end of the waveguides.

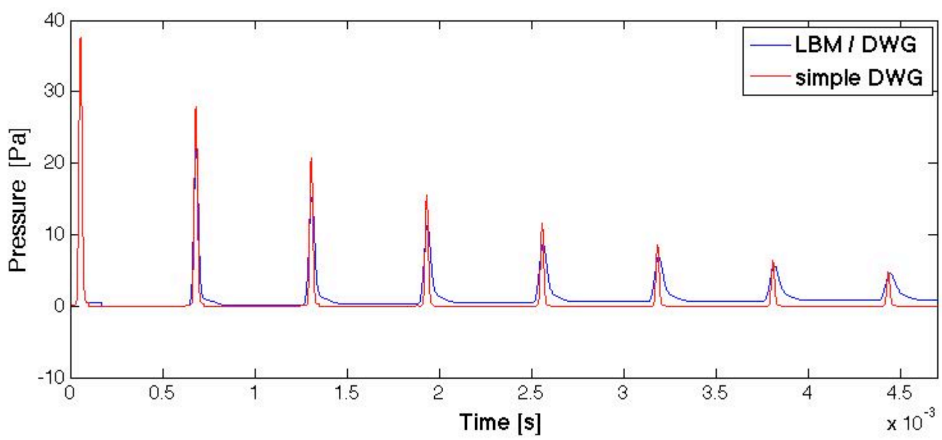

Figure 4: Impulse responses of two closed-closed pipe models.

Figure 4 depicts the impulse responses associated with each system and indicates that they are in phase, meaning that the models have the same effective length. Nevertheless, a significant disagreement can be observed mainly due to the effect of wave dispersion intrinsic to the LBM segment of the first model (Fig. 3-a). This effect is not taken into account by the filter $H(z)$ used in the simple waveguide model. Moreover, the hybrid model tends to a stationary pressure $p > 0$ due to the mass conservation in the system. Conversely, the frequency dependent dissipation implemented by the digital filter used in the simple DWG model does not take into account the conservation of mass.

The Clarinet-like System

Da Silva et al. (2007) developed a single-reed mouthpiece system based on LBM and a finite difference scheme to model the fully-coupled fluid structure interaction between the reed and the flow (Fig. 1). Originally, the system was not coupled to a waveguide and the ABC buffer was set to behave as a non-reflecting boundary. They observed that the original system could achieve self-sustained oscillation when certain pressure differences Δp were applied between the inlet and the outlet of the model. The oscillation was attributed to the modulation of aerodynamic forces during adhesion and detachment of the flow on the reed.

In this paper, the same model is coupled to a digital waveguide, as shown in Figure 2. As a first approximation, a digital filter proposed by Scavone (1999) is used at the end of the waveguide segment to recreate the frequency dependent reflectance of an open unflanged pipe, based on the analytical results derived by Levine and Schwinger (1948). However, this filter representation neglects the eventual influence of the mean flow or vortex shedding at the open end, as reported by Atig (2004).

Figure 5 compares the displacement of the reed measured at the tip for both coupled and decoupled models. The flow is initiated in both models by prescribing a constant inlet pressure $p_m = 5$ kPa.

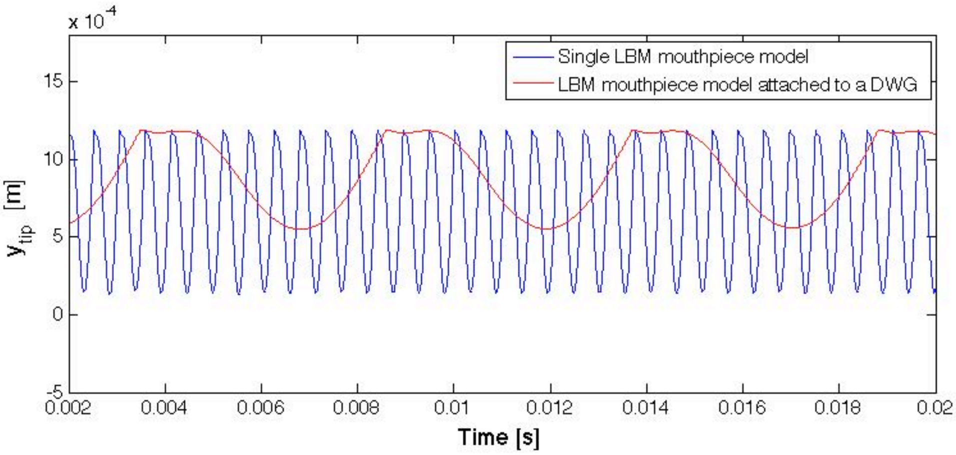

Figure 5: Time histories of the coupled and decoupled LBM mouthpiece-reed model.

In the decoupled case, the reed oscillates very close to its fundamental frequency at free-vibration. Conversely, in the case of the coupled system, the reed oscillates in a much lower frequency, which is very close to the fundamental frequency of the pipe represented by the waveguide component of the system.

SUMMARY

This paper proposed a technique for coupling lattice Boltzmann models to digital waveguides in order to facilitate the representation of wind instruments. This was done by presenting the main aspects of the LBM and DWG theory, the details of the coupling technique, as well as the results for two test cases involving the models of a closed-closed pipe and a single-reed mouthpiece.

The results suggest that the presented technique is able to allow the implementation of any acoustic boundary condition on the lattice domain, as far as the boundary condition can be represented in terms of a digital filter.

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