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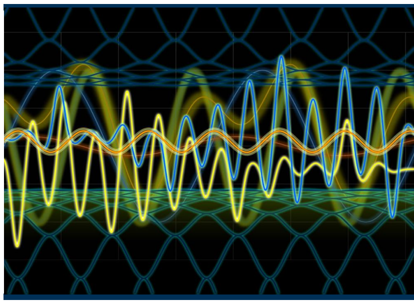

Fourth Vienna Talk on Music Acoustics

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Musical Acoustics: Speed Talk - Woodwinds Session

Acoustical Analysis of the Chinese Transverse Flute (*dizi*) using the Transfer Matrix Method

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The *dizi* is a flute-like traditional Chinese wind instrument with a cylindrical bore and a series of holes opening along its length. In addition to the common embouchure hole and six finger holes, there is a membrane hole located between the embouchure hole and the uppermost finger hole, and four extra toneholes placed near the bottom of the bore, which are always open to the air. In this paper, the transfer matrix method (TMM), as well as the transfer matrix method with external interactions (TMMI), are used to study the acoustic characteristics of the *dizi*. The TMM and TMMI models are validated by comparing the simulated input impedance of the *dizi* with measurements, both with and without a membrane. The TMM is used to generate the distribution map of normalized acoustic pressure and velocity along the main bore as a function of frequency for different fingerings. Different acoustic characteristics are discussed through the analysis of the pressure and flow maps. It is found that the four extra end holes compose a second tonehole lattice, which is independent of the one formed by the finger holes.

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1. INTRODUCTION

The *dizi* is the most popular traditional Chinese transverse flute, usually made of bamboo, and is most distinguished from the western flute by the presence of a hole covered by a wrinkled membrane. The wrinkled membrane is believed to contribute to the unique sound brightness of the *dizi*. Two traditional types of the *dizi* are the *qudi* and *bangdi*.

The general shape of the *dizi* is similar to that of the western flute, as shown in Fig. 1, with a cylindrical bore and a series of toneholes along its length. The membrane hole is located between the uppermost embouchure hole and six downstream finger holes. Four extra end holes are located near the bottom of the bore as follows: two axially distributed front end-holes, like the other tone holes along the front side, as well as two radially distributed back end-holes. Similar components of the end-holes could also be found on the *Xiao*, a Chinese longitudinal flute. From an acoustics point of view, the effective length of the *dizi* is from the cork to the end, without the flute head. The subsequent modeling will also target this part.

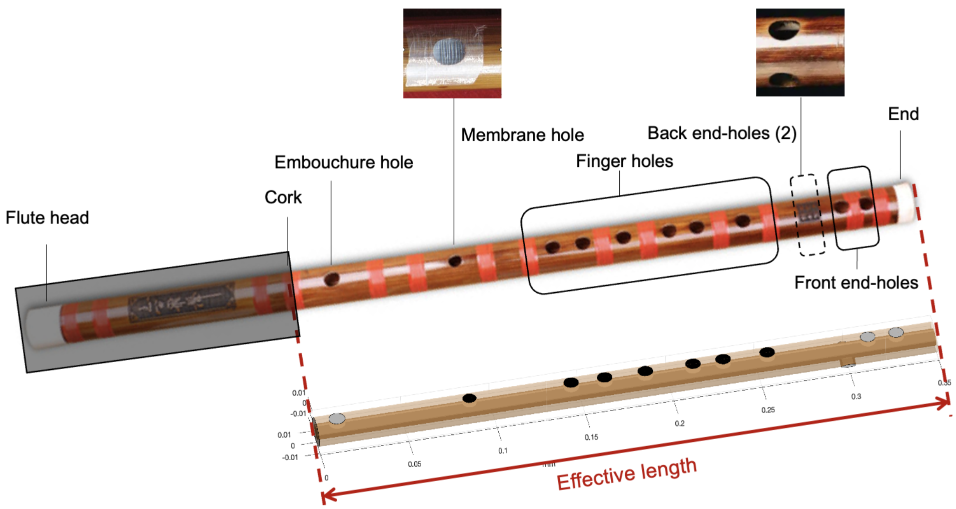

Figure 1: Genral shape of the dizi.

In this study, we first measure the geometry of an F key *bangdi* as well as the input impedance with different fingerings, both with and without a membrane. Then, the transfer matrix method (TMM), as well as the transfer matrix method with external interactions (TMMI), are used to study the acoustic characteristics of the dizi. The TMM and TMMI models are validated by comparing the simulated input impedance of the dizi with measurements. Additionally, the transfer matrices of the three specific components of the *dizi*, the upstream branch, membrane hole and back end-holes are different from the ordinary toneholes, so these expressions are derived. Based on these measurements and modeling results, analyses of the upstream branch, membrane hole and the complete *dizi* are presented. The analyses make use of measurement data when possible, though TMM and TMMI modeling results are used for aspects where measurement data is not available. We try to explain some unique acoustical effects of the *dizi* through analysis of the input impedance and the cut-off frequency of the tonehole lattice, and the sound pressure distribution along the *dizi*.

2. METHODOLOGY

A. MEASUREMENT

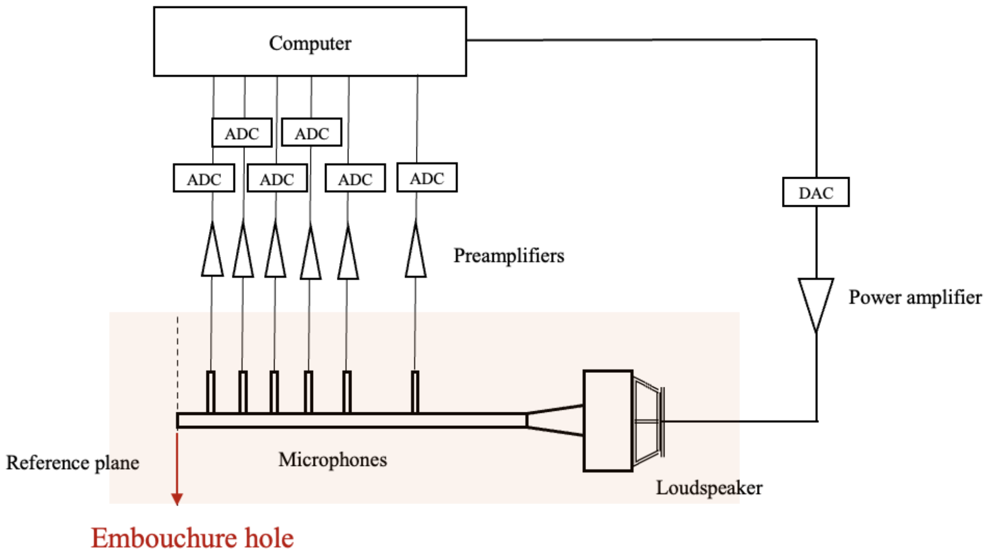

Figure 2: Schematic of the measurement system.

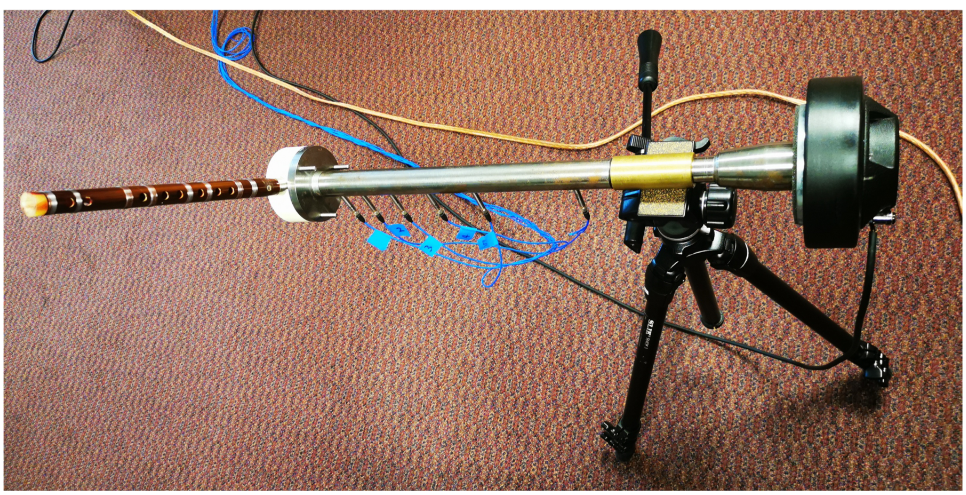

Figure 3: Image of the impedance tube.

A custom-build multi-microphone system based on the least-mean-square signal processing technique is used to measure the input impedance of the *dizi* (see Fig. 2 and Fig. 3) . Six microphones are located along the impedance tube and three non-resonant loads are used to calibrate the apparatus, including a quasi-infinite impedance, an almost purely resistive impedance, and an unflanged pipe radiation load.[1,2] The embouchure hole of the *dizi* is connected to the reference plane of the measurement system by a 3D-printed coupler specially designed, having the same inner geometry as the embouchure hole to ensure a good connection and sealing of the system. The input impedance of each fingering is measured sequentially with and without the membrane. For the condition referred to as “membrane-less” or “without membrane” in the subsequent discussion, the membrane hole is closed. The sealing of the toneholes is achieved by Blu-Tack.

The fingerings corresponding to the measurements consisted of eight groups without half-holes (OXXXXX, XXXXXX, XXXXXO, XXXXOO, XXXOOO, XXOOOO, XOOOOO, OOOOOO) and five groups with half-holes (DOOOOO, XDOOOO, XXDOOO, XXXXDO, XXXXXD), where O stands for open, X for closed and D for semi-closed and the corresponding finger hole sequence starts from the upstream of the *dizi*. Fingerings without half-holes correspond to the first two octaves of the F key, with the diatonic notes C5 to B7. However, the area of the half-hole in the measurement is exactly half the area of the entire hole, whereas the exact size of the hole when using a half-hole technique during the performance is likely different.

B. MODELING

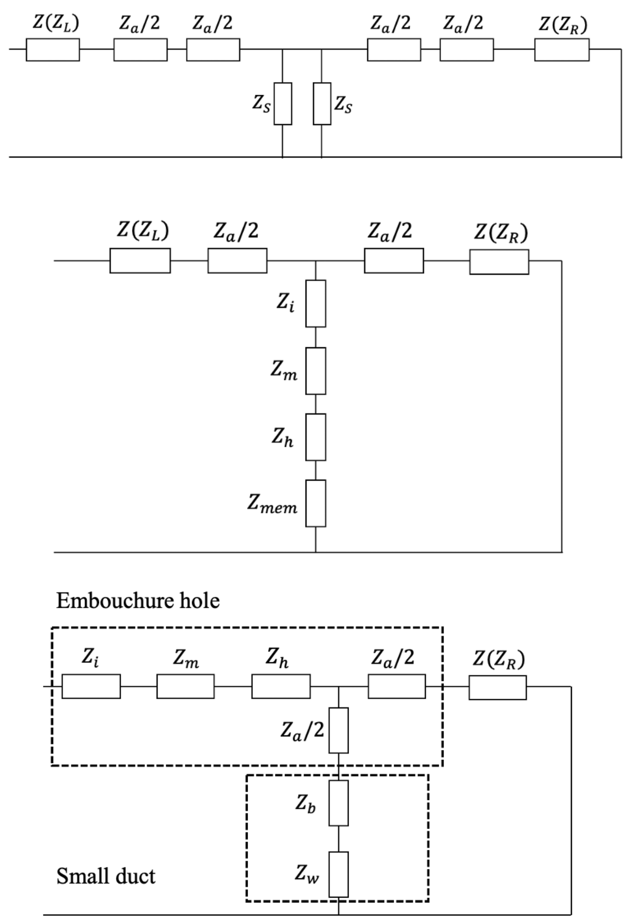

Figure 4: Equivalent circuits representations for the back end-holes (top), membrane hole (middle), and embouchure hole (bottom).

A brief explanation of the TMM is as follows: The wind instrument can be approximated as a series of one-dimensional segments, and each segment can be mathematically represented as a $2{\times}2$ transfer matrix (TM) that relates its input to output frequency-domain quantities of pressure (P) and volume velocity (U), which is complex and frequency-dependent. The input impedance can then be derived by multiplying all the matrices one by one.

The TMMI was developed to more accurately calculate the input impedance by accounting for the external interactions of multiple openings, which is ignored in the TMM and can be expressed as the mutual radiation impedance.[3] As the number of openings increases, the difference between the results obtained by TMMI and TMM increases, especially for frequencies above the tonehole lattice cutoff frequency.

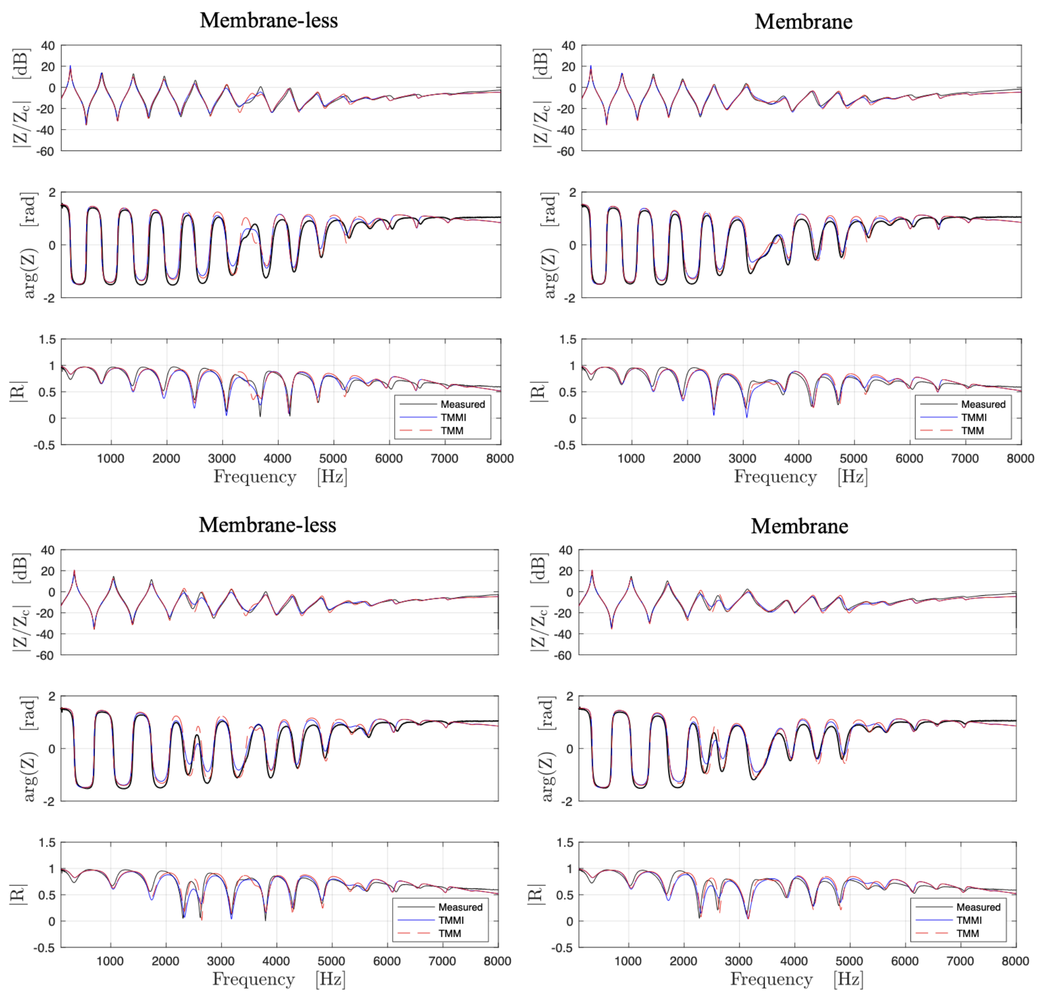

Figure 5: Normalized input impedance modulus and phase and reflection coefficient modulus for the XXXXXX (top) and XXXXOO (bottom) fingering of the dizi. The black curves represent measurement data, the blue curves represent TMMI data and the red curves represent TMM data. The top two graphs correspond to the membrane-less dizi, and the bottom two graphs correspond to the membrane dizi.

The basic tonehole can be approximated as a symmetric T section, and its TM can be presented by an equivalent circuit,[5] which is widely used in wind instrument modeling. The TM of a tonehole is written as[6]

$$\begin{aligned}\mathbf{T}_{hole} &= \begin{bmatrix} 1 & Z_a/2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1/Z_s & 1 \end{bmatrix} \begin{bmatrix} 1 & Z_a/2 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1+\frac{Z_a}{2Z_s} & Z_a(1+\frac{Z_a}{4Z_s}) \\ 1/Z_s & 1+\frac{Z_a}{2Z_s} \end{bmatrix},\end{aligned} \tag{1}$$

where Z_a and Z_s are the series and shunt impedances. The shunt impedance can be subdivided into Z_i, Z_m and Z_h according to the contribution of inner transitional correction, matching volume and chimney height, separately. Since the TM of the basic tonehole model cannot be directly used to model the back end-holes, membrane hole and embouchure hole, Fig. 4 illustrates the equivalent circuits used in our modeling. $Z(Z_L)$ and $Z(Z_R)$ represent the impedances of the rest of the left and right sides. Since the two back end-holes are distributed along the radial direction, they are considered to be connected in parallel. This results in a parallel combination of Z_s terms and two Z_a terms in series on both sides of the shunt impedances. For the membrane hole, another term of membrane impedance Z_{mem} is added to the shunt impedance to model the wrinkled membrane. When the embouchure hole of the *dizi* is considered as the input, the short tube between the cork and the embouchure hole and the downstream main tube are connected in parallel. Therefore, it is necessary to rearrange the impedances. The bottom diagram of Fig. 4 provides an equivalent circuit for this structure. The dashed-line boxes are used to distinguish the embouchure hole from the small duct. Z_b is the impedance of the cylindrical duct and Z_w is the cork terminal impedance, which is approximated as being infinite, corresponding to a rigid wall.

The modeling of the *dizi* using TMM and TMMI in this work is based on G. Scavone's Matlab toolbox "tmmi".[4] The input impedance spectra for fingering XXXXXX and XXXXOO are shown in Fig. 5. As can be seen from the figure, the modeling and measurement results match well, which allows some confidence in the subsequent modeling-based analyses.

3. ANALYSIS OF THE CHARACTERISTIC OF THE DIZI

A. UPSTREAM BRANCH

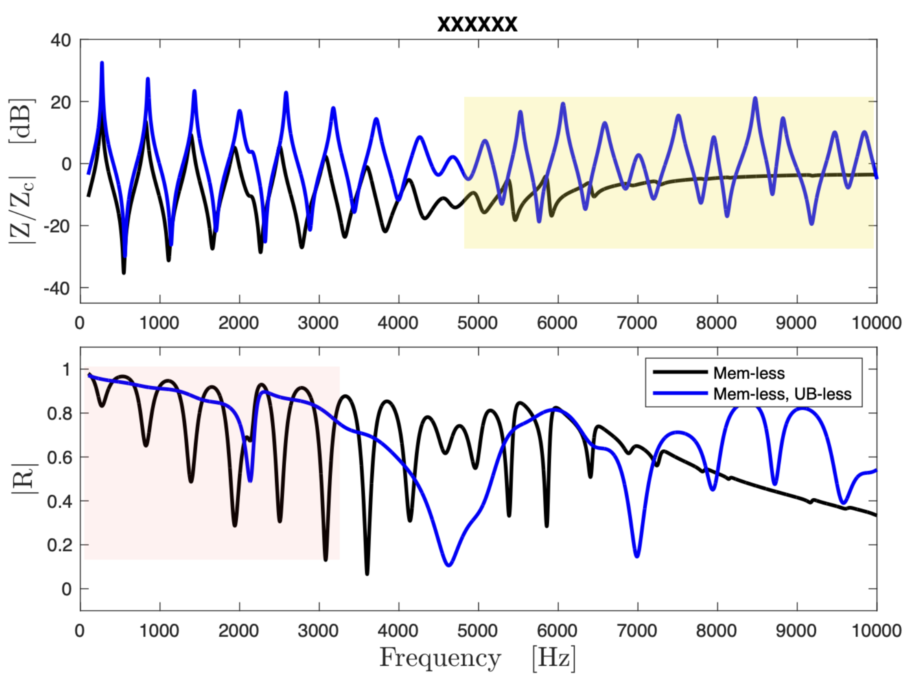

Figure 6: Normalized impedance modulus and reflection coefficient modulus for the XXXXXX fingering of the dizi, calculated using the TMMI. The black curves represent no membrane and the blue curves represent no membrane and no upstream branch.

The upstream branch is the small tube between the cork and the embouchure hole, including the embouchure hole. The input impedance spectrum of the flute becomes suddenly weaker around a certain frequency, which is caused by the upstream branch.[7] This effect is called the Helmholtz shunt and it also occurs for the *dizi*.

Figure 6 compares the input impedance and reflection coefficient curves modeled by TMMI in the case of no membrane, with or without an upstream branch. Compared with the part marked in yellow, the upstream branch leads to a decrease in amplitude after 6kHz, which is the Helmholtz shunt effect.

The cut-off frequency of the tonehole lattice can be estimated through the reflection coefficient.[8] However, due to the upstream branch for flute instruments, the more complex reflectance characteristic makes it more difficult to identify the cut-off frequency, as shown by the black curve in the red part of Fig. 6. Therefore, we suggest that when looking for the cut-off frequency of flute instruments, it is not necessary to consider the upstream branch. It is easy to find that the cut-off frequency is near 2kHz by the first minimum through the reflection coefficient curve for the *dizi* without the upstream branch, corresponding to the blue curve in Fig. 6 below.

B. MEMBRANE HOLE

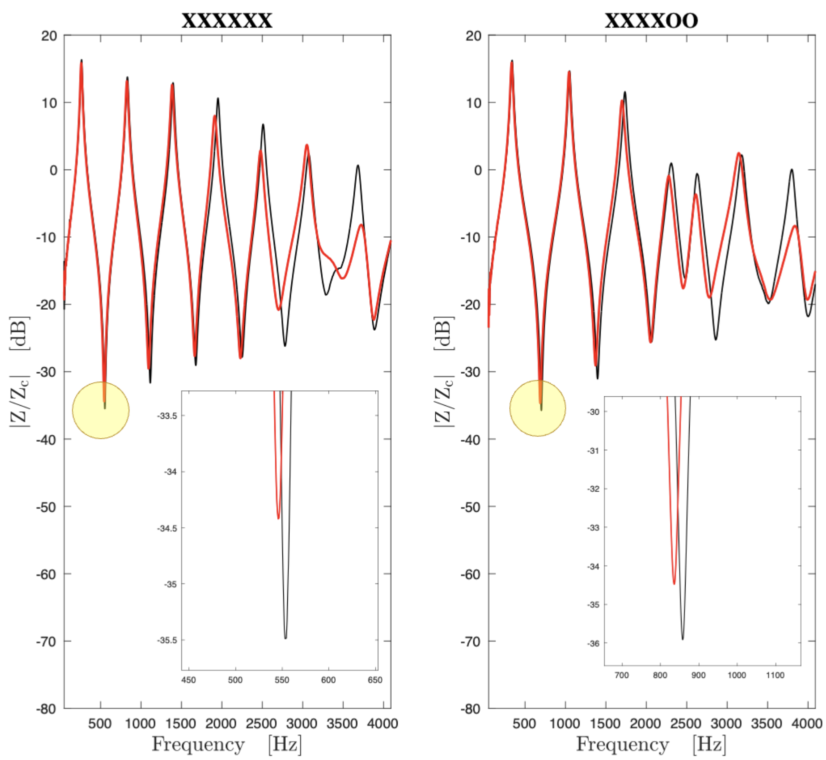

Figure 7: Measured normalized impedance modulus of the dizi, corresponding to XXXXXX fingering on the left and XXOOOO fingering on the right. The black curves represent membrane-less dizi, the red curves represent membrane dizi and the first minima are zoomed in.

As a particular component of the *dizi*, the wrinkled membrane that covers the membrane hole is believed to contribute to the unique sound brightness of the *dizi*. To study the influence of the membrane, the measured input impedance curves with and without the membrane are compared as shown in Fig. 7, corresponding to the XXXXXX and XXOOOO fingerings. Flute instruments are open to the air at the embouchure hole, so they operate at or near the minima of the input impedance. It can be seen in Fig. 7 that the

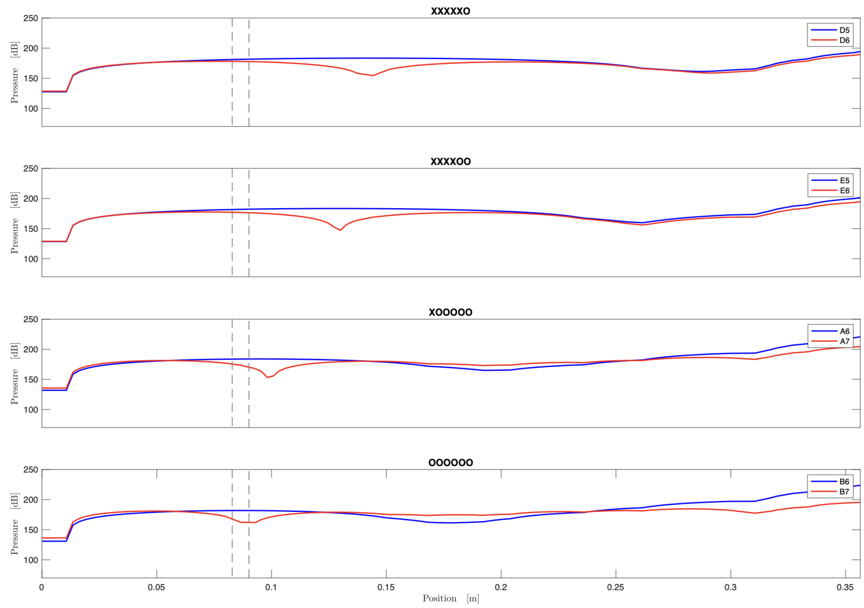

Figure 8: Relative standing pressure wave distribution in the air column of the dizi with the membrane. From top to bottom, the blue curves correspond to the notes D5, E5, A6, and B6 (the first octave), and the red curves for D6, E6, A7, and B7 (the second octave). The abscissa represents the distance from the cork, and the ordinate represents the sound pressure level. The section between the gray dotted lines is the position of the membrane hole.

minimum value increases and the frequency decreases for the first octave when the membrane is attached, which are referred to as "resonance shifts" and "admittance reductions".[9,10] The magnitude of the resonance shift and admittance reduction is related to the standing-wave pressure profile at the membrane hole location. The TMM is used here to generate the pressure standing wave patterns to verify this relationship. A reference value should be defined at first to start calculating the pressure by TMM. In this work, the volume velocity at the input is set to 1. Therefore, by dividing the *dizi* into several small sections (the length interval is less than 1.2cm for each section), the relative pressure of each position can be calculated.

Figure 8 shows the relative pressure wave distribution in the air column of the *dizi* with the membrane for different fingerings. The frequency here is related to the first two minimum selected in the input impedance measurement diagrams for *dizi* without membrane under different fingerings. It can be seen that in the first octave, with the increase of openings, the membrane hole is getting closer to the peak value of the pressure wave, and the magnitude of the pressure has an increasing trend. However, for the second octave, as the number of openings increases, the membrane hole gets closer and closer to the valley of the pressure wave.

To combine the resonance shifts and admittance reductions effect with the sound pressure change at the membrane hole for each fingering, Fig. 9 contrasts various physical parameters. The curves in the top three graphs are from measurement results to ensure more accuracy. From the overall comparison and analysis in Fig. 9, the blue curves for C5-B6 all tend to increase upward, while the red curves for C6-B7 all tend to decrease downward. From this, it can be concluded that there is a positive correlation between the sound pressure level at the membrane hole and the resonance shift and admittance reduction effects. And it can be found that the octave shifts more for fingerings with more openings, resulting in inaccurate octaves of these fingerings.

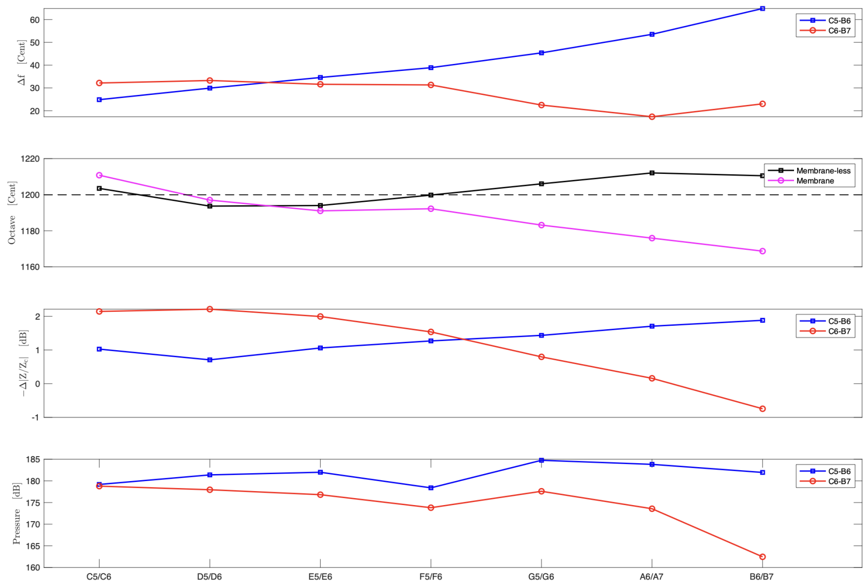

Figure 9: The top graph represents the frequency offset Δf of each note due to the membrane for C5-B6 (blue square) and C6-B7 (red circle), expressed in cents. The second graph shows the frequency intervals for the bottom two octaves with (black square) or without (pink circle) the membrane, expressed in cents. The reference value of 1200 cents, the octaves in equal temperaments are marked with a gray dashed line. The third figure shows the amplitude shifts $-\Delta|Z/Zc|$ due to the membrane for each note within the first octave (blue square) and second octave (red circle). The last graph compares the relative sound pressure at the center of the membrane hole, with C5-B6 represented by blue squares and C6-B7 represented by red circles. Note that the data for the first three graphs are from measured data, while the last graph is calculated using TMM.

C. WHOLE DIZI

For flute instruments, the frequencies near the minima of the input impedance curve are related to the playing pitch, and the depth of the valley is assumed to be related to the playability of the note. The measured input impedance spectra of all fingerings for the experiments are shown in Fig. 10 to analyze the overall characteristics of the *dizi*.

Looking at the top two graphs of Fig. 10, the F key *bangdi* can be divided into three frequency bands according to the characteristics of the input impedance curve: less than 2.3kHz, 2.3 to 6kHz, and greater than 6kHz. In the first register below 2.3kHz, the envelope of the input impedance curve drops relatively slowly, and the resonance becomes weaker with an increase in frequency, which is mainly caused by viscous-thermal losses. The frequency of the second register is 2.3 to 6kHz, and the peak and valley values of the impedance are relatively reduced, which can be explained as the influence of the cutoff frequency of the tonehole lattice,[7] and agrees well with the cutoff frequency results in Section 3.A. In addition, above the cutoff frequency, the effect of mutual radiation impedance is more obvious. The distortion around 3.5kHz is due to the resonant frequency of the membrane. In the third register above 6kHz, the input impedance curve suddenly becomes flat for each fingering. This behavior is characteristic of flute instruments and is related to the Helmholtz shunt effect, as explained in section 3.A.

By extracting the impedance minima values from the middle plot of Fig. 10 (at the red plus signs),

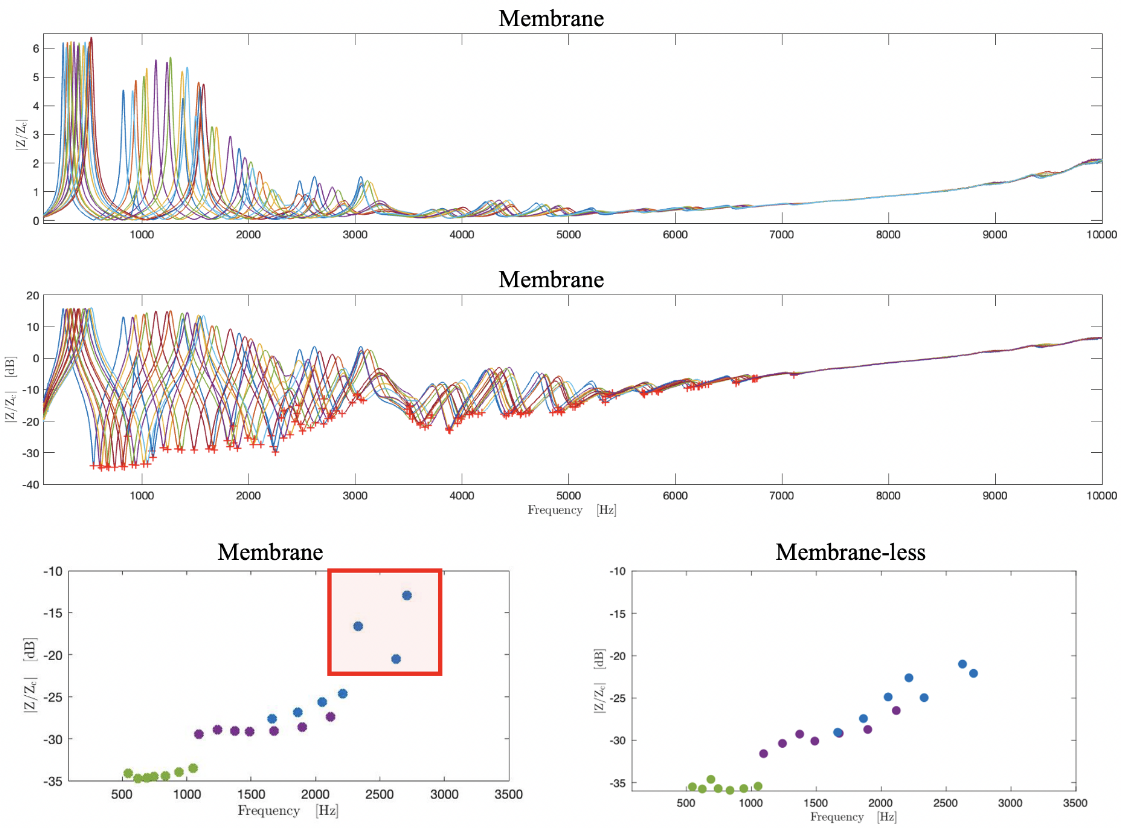

Figure 10: The top two graphs are the input impedance measurement curves of all fingerings for the dizi with membrane, using linear and decibel scales, respectively. The red plus signs are the minimum values obtained by the peak-seeking algorithm. The valleys corresponding to the first three octaves for the second figure are picked out and shown in the bottom scatter diagrams, taking the valley frequency as the abscissa, and the amplitude of the input impedance as the ordinate. The left side represents the dizi with a membrane, and the right side represents without a membrane. The green, purple and blue dots correspond to the notes in the first, second and third octaves, respectively.

the data suggests that the presence of the membrane hole has a detrimental influence on the playability of the highest three notes with the corresponding fingerings XXOOOO, XOOOOO, and OOOOOO (compare the bottom side-by-side plots). The reason for this phenomenon is due to the resonance frequency of the wrinkled membrane.

The pressure standing-wave diagrams discussed in section 3.B can be used to create pressure spectrum maps. This type of map can also be obtained through FEM.[11] The distribution of pressure along the main bore as a response to a unit impulse flow is shown in Fig. 11. The dark line around 8560Hz in the two graphs can be explained by the resonance of the upstream branch. The blue box area from 13 to 17kHz in the graph above is a region of rejection, which is caused by the closed tonehole lattice. Thus, as shown in the bottom figure, if all toneholes are open, the blue area disappears.

To find the overall characteristics of the *dizi*, now add the normalized pressure for the fingerings from XXXXXX to OOOOOO together, as shown in Fig. 12. The cut-off frequency can be found directly by noting the frequencies of horizontal color mutations, as pointed out by the red arrow in Fig. 12 near 2260 Hz. This result is consistent with the result found in section 3.A, through the reflection coefficient curve of the *dizi* without the upstream branch. By contrasting the top two maps, it can be found that the overall

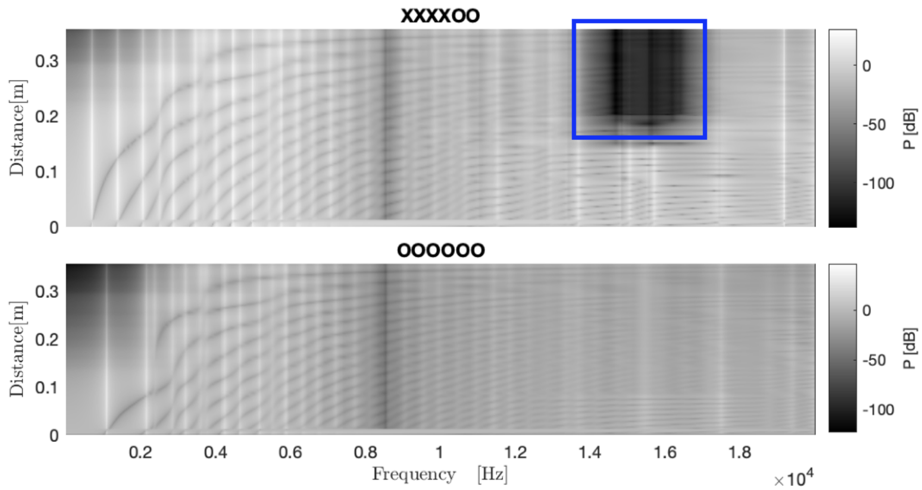

Figure 11: Normalized pressure map of dizi without membrane. The horizontal axis shows the frequency, and the vertical axis represents the position along the bore relative to the cork. The top is the result of XXXXOO fingering and the bottom is for OOOOOO fingering.

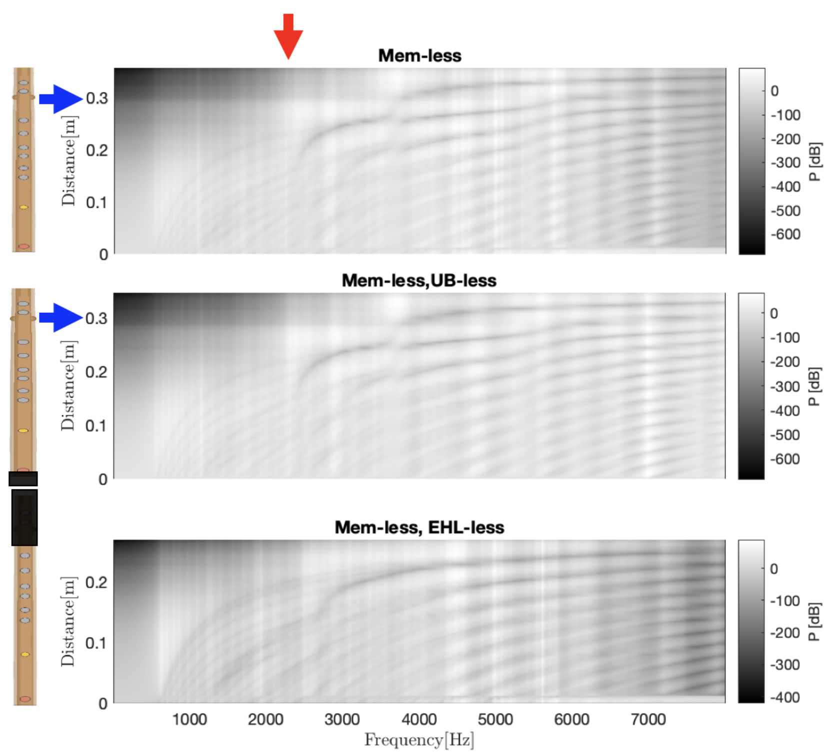

Figure 12: Normalized added pressure map of dizi without membrane for fingerings from XXXXXX to OOOOOO. The upper map is for the whole dizi, the middle is for the dizi without the upstream branch, and the lower is for the dizi without the EHL.

performance is not changed too much below 6kHz.

There are vertical color mutations at the positions corresponding to the blue arrows in these two maps, which are related to the gaps between finger holes and end-holes from the geometrical structure. This phenomenon is also shown in Fig. 11. It indicates that the end-holes that have been open all the time will affect the characteristics of the *dizi*. Therefore, the toneholes of the *dizi* can be divided into two lattices: finger hole lattice (FHL) and end hole lattice (EHL). As shown in the bottom figure, when cutting off the EHL, the vertical color mutations disappear.

4. CONCLUSION

In this study, we present an acoustical analysis of the F key *bangdi* based on measurements and TMM / TMMI modeling. To model the *dizi*, the equivalent circuit of the back end-holes, membrane hole, and embouchure hole are given here. Based on the measurement and modeling results, some acoustical analyses of these particular aspects of the *dizi* are provided. We suggest that when looking for the cut-off frequency of the flute instruments, it is not necessary to consider the upstream branch. By contrasting the standing pressure wave and the impedance spectrum, it is confirmed that the membrane's location is related to the degree of resonance shift and admittance reduction effect. We try to explain some unique acoustics effects of the *dizi* by analyzing some physical parameters such as the input impedance and the cut-off frequency of the tonehole lattice. When analyzing the characteristic of the whole *dizi*, some effects are noted, such as the resonance of the membrane and upstream branch, and the rejection region caused by a closed tonehole lattice. Through the normalized pressure map, it is found that the toneholes can be divided into FHL and EHL.

REFERENCES

[1] P. Dickens, J. Smith, and J. Wolfe, "Improved precision in measurements of acoustic impedance spectra using resonance-free calibration loads and controlled error distribution," *J. Acoust. Soc. Am.* **121**(3), 1471-1481 (2007).

[2] J. A. Kemp, A. Jonathan, M. van Walstijn, D. M. Campbell, J. P. Chick, and R. A. Smith, "Time domain wave separation using multiple microphones," *J. Acoust. Soc. Am.* **128**(1), 195-205 (2010).

[3] A. Lefebvre, G. P. Scavone, and J. Kergomard, "External tonehole interactions in woodwind instruments," *Acta Acustica united with Acustica.* **99**(6), 975-985 (2013).

[4] G. Scavone, "A tutorial on the transfer matrix method for acoustic modeling," *J. Acoust. Soc. Am.* **149**(4), A93-A93 (2021).

[5] C. J. Nederveen, J. K. M. Jansen, and R. R. Van Hassel, "Corrections for woodwind tone-hole calculations," *Acta Acustica united with Acustica.* **84**(5), 957-966 (1998).

[6] D. H. Keefe, "Theory of the single woodwind tone hole," *J. Acoust. Soc. Am.* **72**(3), 676–687 (1982).

[7] J. Smith, J. Wolfe, and M. Green, "Head joint, embouchure hole and filtering effects on the input impedance of flutes," *Proceedings of the Stockholm Music Acoustics Conference.* 295-298 (2003).

[8] E. Petersen, V. Turcotte, and T. Colinot, "Objective Metrics to Identify the Tonehole Lattice Cutoff Frequency of Conical Woodwind Instruments," *Forum Acusticum.* 3177-3180 (2020).

[9] C.-G. Tsai, "The Chinese membrane flute (dizi): physics and perception of its tones," Ph.D. thesis, Humboldt-Universitat Zu Berlin. (2003).

[10] H. H. Ng, D. Y. Tan, T. S. Ng, and J.-M. Chen, “An acoustic impedance analysis of the dizi (Chinese transverse flute),” *ICSV27.* (2021).

[11] E. A. Petersen, T. Colinot, F. Silva, and V. H.-Turcotte, “The bassoon tonehole lattice: Links between the open and closed holes and the radiated sound spectrum,” *J. Acoust. Soc. Am.* **150**(1), 398-409 (2021).