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CLAMPED BAR MODEL FOR FREE REEDS

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ABSTRACT

Previous models for the synthesis of free-reed instrument sounds have considered the oscillating free reed as a damped spring-mass system. Our recent experiments to study the reed motion using a laser vibrometer suggest that the oscillating reed shows harmonics which are more prominent at higher excitation pressures. To better characterize this behavior, we present a distributed model of the free reed using a fourth-order partial differential equation where the reed is modeled as a uniform bar, clamped at one end. A numerical scheme is developed for the time domain simulation of the reed motion and the air flow. The spectral characteristics of the simulated reed motion are studied and agreement with the experimental measurements is discussed.

Keywords: *free reed acoustics, distributed reed model, musical acoustics, laser vibrometer*

1. INTRODUCTION

Western free reed instruments such as the harmonium, the accordion and the harmonica are a relatively new category of musical instruments having originated in Europe in the 19th century. Cottingham [1] has described the sound production mechanism in free reed instruments and reviewed the prominent acoustic studies of this category of musical instruments.

Self-sustained oscillations can be set up in reeds if an unsteady pressure difference can be developed between the two sides of the reed. As the reed oscillates, the aperture area changes, leading to velocity fluctuations in the air flow through the reed. The velocity-dependent pressure changes can be found using the Bernoulli equation. However, this mechanism in isolation cannot develop the asymmetric pressure differences required to set up self-oscillations [2]. In highly damped reeds with strong coupling with the resonator as in the case of a clarinet, the pressure asymmetry is provided by the acoustic response of resonator, forcing the reeds to oscillate near the fundamental modes of the resonator. Free reeds in contrast are lightly damped and tend to oscillate close to their natural frequencies.

St. Hilaire [3] first proposed that the inertial effect of the upstream air flow was the primary mechanism to set up self-oscillations in free reeds. Since the air flow in the reed happens through a narrow jet, there is much less mass in the jet as compared to the upstream section. As a result the pressure near the reed changes rapidly as compared to the upstream pressure. St. Hilaire's model showed that there was a component of the pressure near the reed changing in phase with the reed velocity, thus leading to a net positive power transfer to the reed. Experimental studies by Tarnopolsky [4, 5] and Ricot [6] agree with the upstream inertial effect theory proposed by St. Hilaire.

Further studies by St. Hilaire [7, 8] have described the nonlinear mechanism which leads to a limiting amplitude for the reed oscillations and the presence of harmonics in the sound generated.

Millot and Baumann [9] provided a minimal model for the synthesis of any free reed instrument. They also described a numerical scheme to solve the system in real time and presented a discussion on the stability of the numerical scheme and the dependence of the playing frequency (pitch) and loudness of the synthesized sound.

In a previous work [10], the authors made suitable adaptations to the Millot-Baumann model to match the playing parameters and timbre of an Indian harmonium with hand bellows. The main feature of this model was the use of a source-filter structure for the synthesis. The source was modeled as 1D physics-based model of a free reed interacting with the air flow. The effect of the wooden enclosure of the instrument was approximated by an all-pole filter whose coefficients were estimated from a recorded harmonium sound. The model assumed the reed to be a sinusoidally oscillating lumped mass-spring-damper whose behavior is predominantly governed by the eigenfrequency and the quality factor parameters.

Subsequently, experiments were conducted with a laser vibrometer to study the motion of free reeds mounted on a mechanical blower. The experimental measurements suggested that reed undergoes transverse oscillations which show harmonics, especially when the reed is excited with a higher pressure. With an aim to characterize this behavior, we developed a 1D distributed model of the free reed modeled as a Euler-Bernoulli beam.

Section 2 describes the results from the laser vibrometer experiments. Section 3 describes the clamped bar model and numerical scheme implemented. In section 4, we discuss the agreement of the clamped bar model with the measurements and possible improvements.

2. LASER DOPPLER VIBROMETER MEASUREMENTS

Figure 1: Assembly for Laser Vibrometer measurements.

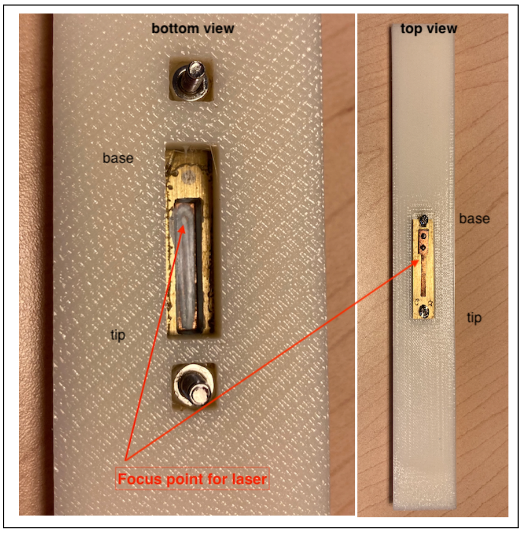

Figure 2: Harmonium reed attached to a 3D printed mount with a slot. Point of focus for the laser is shown

The motion of a B5 (990 Hz) harmonium reed was studied using a Laser Doppler Vibrometer (LDV). For this purpose, a slot to hold the reed was 3D printed and was mounted on a mechanical blower assembly made from aluminum and fiber glass. Observations were recorded while air was blown into the mechanical blower through a pipe by mouth with different pressure patterns that were sufficient to produce a loud sound from the reed. The laser beam from a Polytec PDV-100 vibrometer was focused at a point closer to the base (shown in Fig. 2). The focusing point was chosen such that the range of the vibration velocities observed was within the measurement range of the LDV ($\pm 0.5 m/s$). The pressure inside the mechanical blower was simultaneously measured using a pressure transducer (Endevco 8510B) and signal amplifier (Endevco Model 136). To record the vibration velocity and pressure signals a National Instruments USB-4431 Signal Acquisition Board was used. Fig. 1 shows the assembly used during the measurements. From our previous experiments, it was known that the reed chamber pressures in a real harmonium are in the range of $0.2 - 1.5\ kPa$. A similar range of mouth pressures was observed in the measurements using the mechanical blower. Significant observations from the velocity measurements by the LDV are discussed next.

In Fig. 3 and Fig. 4 respectively, the log-magnitude

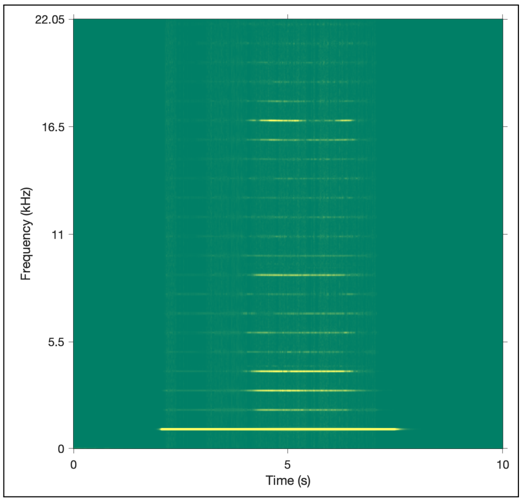

Figure 3: Log-magnitude spectrogram for reed velocity in an LDV measurement. Higher harmonics are prominent at higher blowing pressure.

spectrogram of the reed velocity and the corresponding mouth blowing pressure for a reading are presented. During this reading, the blowing pressure was initially low so that the sound produced by the reed was barely audible. The reed was then blown as strongly as possible to produce a loud sound. It can be seen that at lower blowing pressures (≈ 0.2 kPa) the motion of the reed is largely sinusoidal while the higher harmonic components become more prominent at the higher blowing pressures (1.4-1.6 kPa).

3. CLAMPED BAR MODEL

In previous work [10], the Millot and Baumann minimal model [9] was adapted to provide a better match for the playing parameters and the timbre of an Indian hand harmonium. While our lumped model produced quite convincing synthesis, some model parameters had to assume unrealistic values for a numerically stable synthesis. For example, the thickness of the reed would be several times that of its length and its width. In the synthesis using this model, the reed practically had a sinusoidal motion. Hence, the peculiar spectral properties observed experimentally and described in Section 2 were not observed in the synthesis.

To address these issues, a 1D distributed model of the reed was attempted. A distributed reed modeling approach has been previously adopted by Stewart and Strong

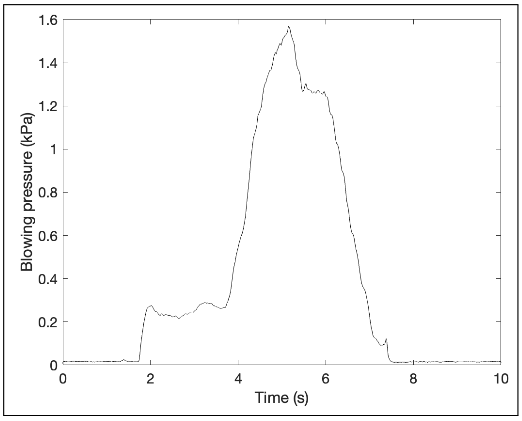

Figure 4: Blowing pressure during the reed velocity measurement in Fig. 3

[11] and Avanzini and Walstijn [12] for modeling clarinet reeds. Sommerfeldt and Strong [13] used the model from [11] to develop a time domain player-clarinet simulation system. These approaches modeled the reed as an Euler-Bernoulli beam fixed at one end free to move at the other, which is also the approach that we used.

3.1 Model description

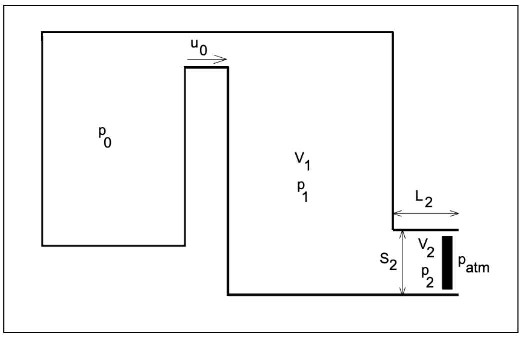

Figure 5: Configuration of the lumped physical model used in [10].

Fig. 5 shows a schematic representation of the configuration of the lumped physical model used to simulate the harmonium free-reed in [10]. The model assumes the region upstream of the reed (i.e. the reed chamber) to be a large volume V_1 where the pressure p_1 is uniform. The volume V_2, with cross-sectional area S_2 and length L_2, models the region near the reed that supplies air to the reed. The reed itself was modeled as a sinusoidally oscillating lumped mass-spring-damper whose behavior is predominantly governed by the eigenfrequency (ω_0) and the quality factor (Q) parameters. The region downstream from the reed is exposed to atmospheric pressure. In the original Millot-Baumann model [9], the system was excited by the volume flow u_0 entering the volume V_1. In our adaptation, however, we added the chamber with pressure p_0 to represent the bellows pressure which indirectly controls the excitation signal u_0. This change led to a better agreement between experimentally measured reed-chamber pressures and the corresponding p_1 values in the synthesis. It also allowed for the use of bellows pressure as a parameter to control the sound produced, like in a real harmonium.

The governing equations for the lumped physical model of the reed from [9, 10] are as follows:

- Mass conservation for volume V_1

$$\frac{V_1}{c_0^2} \cdot \frac{d(p_1 - p_{atm})}{dt} = \rho_{air}(u_0 - u) \qquad (1)$$

- Dynamic Bernoulli equation for volume V_2

$$p_1 = p_2 + \rho_{air} \cdot \frac{L_2}{S_2} \cdot \frac{du}{dt} \qquad (2)$$

- Bernoulli equation for the upstream and downstream of the reed

$$p_2 = p_{atm} + \frac{1}{2}\rho_{air} v_j^2 \qquad (3)$$

- Equation of the reed motion as a lumped mass-spring-damper

$$\frac{d^2\zeta}{dt^2} + Q^{-1}\omega_0 \frac{d\zeta}{dt} + \omega_0^2 \zeta = \mu(p_2 - p_{atm}) \qquad (4)$$

- Total volume flow rate as the sum of jet flow and pumped flow due to reed motion

$$u = S_r \cdot \frac{d\zeta}{dt} + \alpha S_u v_j \qquad (5)$$

where,

ζ : displacement of reed from equilibrium position

α : Factor to account for vena-contracta and the additional aperture area from the sides of the reed

S_u : useful area of the reed aperture $= W_{reed} \cdot \sqrt{(y + \Delta y)^2 + h^2}$, Δy being the initial gap between the reed tip and the support plate

S_r : area of the reed

μ : coefficient dependent on the area and the mass of the reed

c : speed of sound in air

v_j : particle velocity in air jet

ρ_{air} : air density

ω_0 : eigenfrequency of the lumped reed

Q : quality factor

u : Volume flow rate of air escaping through the reed aperture

For the distributed model, we replaced the lumped reed equation (Eq. 4) from the system of equations with the dynamic Euler-Bernoulli beam equation given by:

$$\rho_r A_r \frac{\delta^2 y}{\delta t^2} + R\frac{\delta y}{\delta t} + EI\frac{\delta^4 y}{\delta x^4} = S_r \cdot (p_2 - p_{atm}) \qquad (6)$$

where,

y : displacement of reed

ρ_r : density of the reed

A_r : cross section area of the reed

R : damping parameter

E : Modulus of elasticity (Young's Modulus)

I : Bending Moment of inertia

For simplicity, the reed was assumed to have a uniform width and thickness across its length. While Avanzini and Walstijn also included material damping terms in [12], we chose to neglect them since it is known the brass reeds used in harmonium are lightly damped unlike the cane reeds of a clarinet.

The clamped-free boundary condition was imposed by the constraints:

$$y(0,t) = \frac{\delta y}{\delta x}(0,t) = \frac{\delta^2 y}{\delta x^2}(L_{reed},t) = \frac{\delta^3 y}{\delta x^3}(L_{reed},t) = 0 \qquad (7)$$

3.2 Numerical Simulation

To numerically solve the reed equation (Eq. 6), the implicit scheme described by Chaigne and Doutaut [14] and adopted by Avanzini and Walstijn [12] was used. The reed displacement at the current time step was calculated on the basis of reed pressure p_2 at the previous time step. To discretize the rest of the equations (Eqs. 1, 2, 3, 5) the derivatives were replaced by a backward finite difference. A time sampling rate of 4×44100 Hz was used while the reed was approximated as a grid with $N = 40$ sections.

Table 1: Parameters used for the simulations.

Young's modulus	$E = 125$ GPa
Length of reed	$L_{reed} = 16$ mm
Width of reed	$W_{reed} = 2$ mm
Thickness of reed	$t_{reed} = 0.4$ mm
Damping parameter	$R = 0.065$
Reed chamber volume	$V_1 = 0.0013\ m^3$
Length of near-field region	$L_2 = 50\ mm$
C.S. Area of near-field region	$S_2 = 6.4 \times 10^{-5}\ m^2$
Density of air	$\rho_{air} = 1.1769\ kg/m^3$
Density of reed	$\rho_r = 8490\ kg/m^3$
Initial gap between reed tip and support	$\Delta y = -0.1$ mm

Similar to Avanzini and Walstijn [12], convergence of the numerical solution was verified for the discretization parameters. A higher number of grid points were not found to appreciably change the the solution. The other parameters used for the simulation are listed in Table 1. The system of equations was solved using a Newton-Raphson scheme described by Millot in [9].

4. RESULTS AND DISCUSSION

The time-domain waveforms of the reed velocity growth observed in the vibrometer measurement and in a clamped-bar model simulation with bellows pressure of 250 Pa are shown in Fig. 6 and Fig. 7 respectively. It can be verified that the initial exponential growth and the attainment of a limiting amplitude observed experimentally is also present in the clamped-bar model simulation.

The spectrograms of the reed velocities obtained in the simulations with low (250 Pa) and high (1650 Pa) bellows pressures are shown in Fig. 8 and Fig. 9 respectively. The figures confirm that in the simulation, the reed exhibits predominantly sinusoidal motion at low bellows pressures while the presence of higher harmonics is apparent at high bellows pressures. In this aspect, the simulation has a good agreement with the measured data.

Fig. 10 shows a comparison of the normalized Long Term Average Spectrums (LTAS) of the measured reed

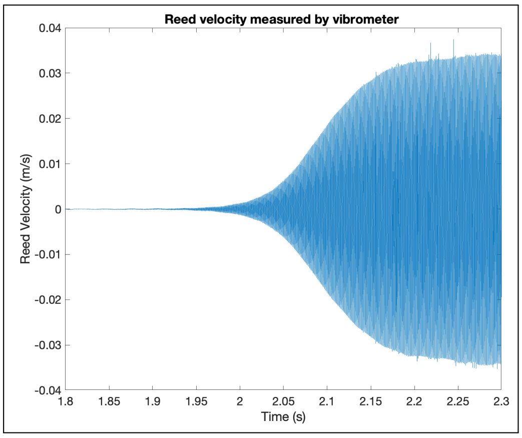

Figure 6: Waveform of reed velocity measured by vibrometer at mouth pressure ≈ 200-250 Pa.

velocity and with the reed velocity in a simulation with high excitation pressures. The magenta curve shows the normalized LTAS of the reed velocity in a simulation of the reed excited with an impulse. The reed response in the simulation (blue curve) shows a formant like structure with stronger peaks near the natural frequencies of the reed. This suggests that the effect of the reed could be to filter the driving signal, i.e. the reed pressure (p_2). The reed pressure (p_2) that drives the reed has a harmonic spectrum due to the non-linear flow mechanism described in equation 3. The reed response in the measurement (cyan curve) has stronger peaks near the fundamental ($\approx\ 990Hz$) and the third natural frequency ($\approx 16000Hz$). However, a clear formant is not visible at the second natural frequency ($\approx 6000Hz$). This deviation could be caused due to a combination of multiple factors. The real reed has some non-uniformity in its thickness but the simulation assumes a uniform reed. There could be other vibrational modes such as the torsional modes [15] that are significant at the operating conditions. The deviation can also be caused by differences in the real and simulated reed pressure which drives the reed.

An additional positive outcome of using the distributed model is that it leads to the fundamental frequency being determined by the measured geometric properties (length, width and thickness) and the material properties of the reed in contrast with the lumped model which uses rather unrealistic parameters. However, the model still uses arbitrary parameters for the dimensions of the far field (volume V_1) and the near field (area S_2 and

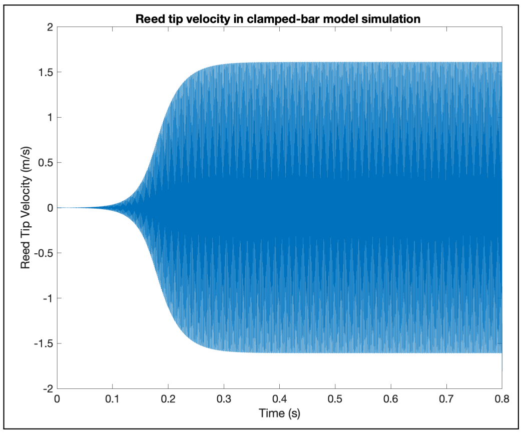

Figure 7: Waveform of reed tip velocity in clamped-bar model simulation with bellows pressure of 250 Pa.

length L_2).

5. CONCLUSION

A clamped bar model for free reeds is presented. The model provides a significant refinement over the lumped mass-spring-damper models used previously. Experimentally observed spectral characteristics of the free reed closely resemble the spectral characteristics in the simulations with the clamped bar model. As in a real free reed, the playing frequency in the clamped model is determined by the measured geometric properties instead of an arbitrarily specified eigenfrequency in the lumped models. Further refinements to the clamped model are possible by assuming a reed with non-uniform thickness and including other vibrational modes. The flow model could be further improved to reduce the arbitrariness in the assumptions of the near and far field dimensions.

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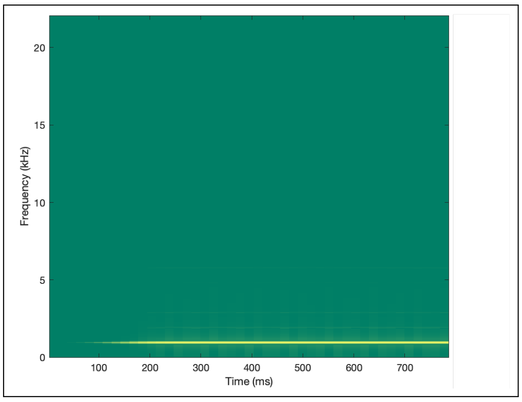

Figure 8: Spectrogram for reed velocity simulation with bellows pressure of 250 Pa.

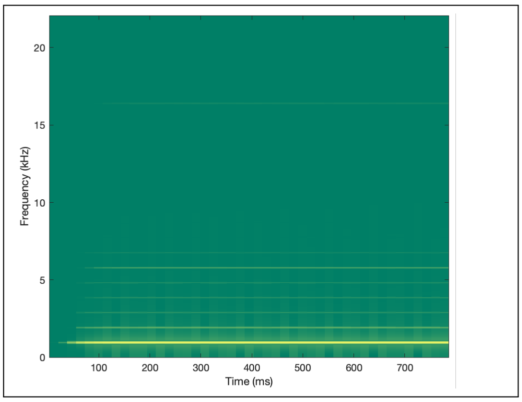

Figure 9: Spectrogram for reed velocity simulation with bellows pressure of 1650 Pa.

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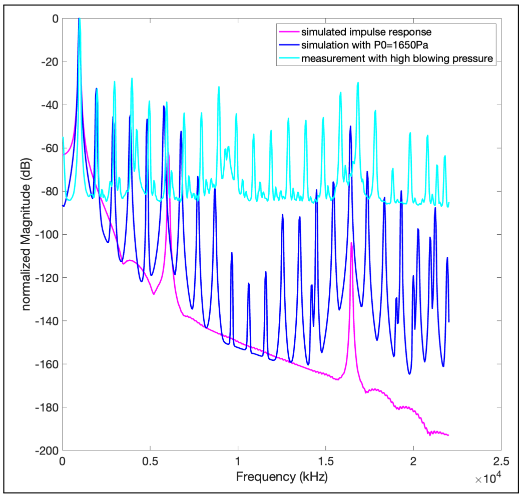

Figure 10: Comparison of normalized Long Term Average Spectrums (LTAS) for simulated reed velocity and measured reed velocity at high excitation pressures. The magenta curve shows the magnitude of the transfer function of the system obtained by exciting the reed with an impulse and computing the LTAS.

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