A Survey of Physical Modelling Techniques for Synthesizing the Classical Guitar

Cory McKay
Faculty of Music, McGill University
555 Sherbrooke Street West
Montreal, Quebec, Canada H3A 1E3
cory.mckay@mail.mcgill.ca

ABSTRACT

This paper presents an overview of physical modelling techniques that have been used to model classical guitars. Areas that still need improvement are discussed. The introduction provides a brief discussion of the motivations for using physical modelling.

INTRODUCTION

Physical modelling synthesis uses mathematical models of physical instruments to synthesize their sound. Although accurate models of the often complex mechanical and acoustic behaviour of instruments can be difficult to construct and expensive to process, this approach does offer a number of important benefits that other synthesis techniques do not.

Perhaps the most important of these benefits is the ability to represent musical events through parameters based on physical configurations and gestures. This allows a level of control that is natural and easy for a musician to learn. Parameterized data can also be easily edited to correct mistakes or introduce desired changes after a recording of performance parameters. Altering the parameters of an instrument itself can also have very interesting effects. It becomes possible to change an instrumental setting to see, for example, how a classical guitar performance might have sounded on a steel guitar. A more adventurous approach would be to simulate types of instruments that would be physically difficult or impossible to construct or play.

The ability to parameterize performance gestures also enables musical data to be stored in a potentially more compact form than raw audio data. When combined with good physical modelling systems, sufficiently complete parameters could make it possible to generate sound that could be perceptively indistinguishable from the audio information that was produced during the original performance.

One approach to collecting parameters for physical modelling is to design original musical controllers that provide as much, or perhaps more, control as the instrument being modeled. Recent research has also made it possible to collect increasingly sophisticated control information from audio signals of acoustic instruments. Traube and Smith (2000), for example, have developed techniques for recovering plucking points and the fingering points from audio recordings of acoustic guitar music. Tolonen and Valimaki (1997) have shown that it is possible to extract the physical parameters of plucked string instruments from their audio signals. The increasing availability of parameters increases the utility of physical modelling systems. One caveat, however: recovering parameters from audio signals is much more difficult when polyphonic music is considered.

The modelling of classical guitars is an area where some success has been obtained. The behaviour of plucked strings is relatively well understood, although only moderate headway has been made in modelling the behaviour of the resonator.

Some of the most important performance parameters that characterize guitar performances are plucking position, plucking velocity, direction of plucking relative to the strings, duration of pluck, material used for plucking, fingering position, magnitude of fingering force, direction of fingering force, material used for fingering and velocity with which a string is pressed and released. Some of the most important parameters regarding the instrument include length of the strings, tension of the strings, mass per unit length of the strings, rigidity of the strings, rigidity of the nut, elasticity of the bridge, fret position, fret material and coupling between the strings, bridge and body of the guitar. The exact behaviour of the body of the guitar is not fully understood, although parameters such as size, shape and material can be used to grossly model it. The acoustics of the room as well as air pressure and humidity must also be considered. The large number of parameters makes it clear that some assumptions and simplifications are necessary in order to model guitars, at least initially.

From a higher-level perspective, a physical model should be able to accommodate certain common techniques. Plucking position is a particularly important parameter in controlling timbre, as is plucking material (e.g. nails, pick, flesh of fingers). The use of different fingerings for the same notes and chords is also well known to have an important effect on timbre. Muting and the excitation of natural harmonics are also important techniques. Although less attention has been paid to them in the literature, left-hand techniques such as bends, different types of vibrato, hammer-ons, pick-offs, trills, slides (both with fingers and with slides) and tapping are also an essential part of performance technique.

It becomes clear from all of this that there is great deal to consider if one wishes to accurately model an acoustic guitar. This paper will present an overview of the work that has been done to date on plucked string modelling in general and guitar modelling in particular, and will point out areas that still need improvement.

BASIC KARPLUS-STRONG SYNTHESIS

Karplus-Strong synthesis offers a simple and efficient method for generating signals that sound similar to plucked strings. Although it was first arrived at by accident rather than through an intentional attempt to model the physical properties of plucked strings, it has since been shown to have a physical basis (see below).

In its simplest form, the Karplus-Strong algorithm involves the use of a re-circulating delay line with a filter placed in the feedback loop (see Figure 1). Sending a stream of n random samples into the system initiates a note. The random samples then circulate through the feedback loop. Since there is a fixed pattern of length n, this results in a periodic signal, even though the samples themselves are random. The resulting signal therefore has a frequency of R / n, where R is the sampling rate.

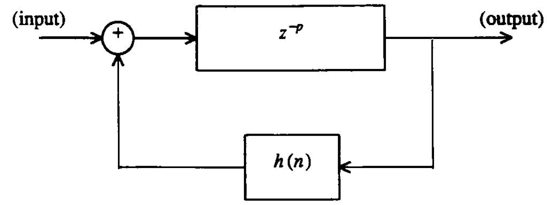

Figure 1: Basic implementation of the Karplus-Strong algorithm. The box labelled z^{-p} delays all input by p samples and the box labelled *h(n)* is a digital filter (Moore, 1990).

If the filter is a low-pass filter, then the higher frequency components will become increasingly attenuated as the samples continue to cycle through the system. If the gain of the filter is less than one at the fundamental frequency, the result will be a signal that is initially harmonically rich, but then decays to a signal close to a simple sinusoid. This behaviour is very similar to that of a plucked string. A filter with the following impulse response and transfer function could be used to accomplish this (Moore 1990):

$$y(n) = \frac{x(n) + x(n-1)}{2} \qquad (1)$$

$$H(z) = \frac{1 + z^{-1}}{2} \qquad (2)$$

Changing the delay time can be used as a simple technique for altering the pitch of the tone. Of course, it is also necessary to be able to control note duration. A loss factor ρ can be introduced into the filter to shorten note durations:

$$y(n) = \rho \frac{x(n) + x(n-1)}{2} \qquad (3)$$

A stretch factor S can also be incorporated so that high frequencies are attenuated to a lesser degree if a note needs to be lengthened. A filter that incorporates lengthening as well as shortening of notes has the following impulse response and frequency dependant gain:

$$y(n) = \rho\left[(1-S)x(n) + Sx(n-1)\right] \qquad (4)$$

$$G(f) = \rho\sqrt{(1-S)^2 + S^2 + 2S(1-S)\cos(2\pi f / R)} \qquad (5)$$

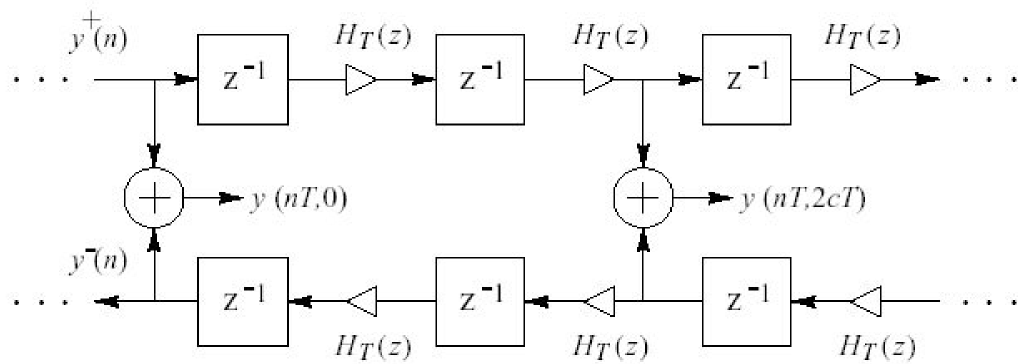

Figure 2: Implementation of an ideal, linear, lossy and dispersive digital waveguide (Smith, 1993).

Unfortunately, this results in a frequency dependant phase:

$$pha\left[H(e^{j\omega})\right] = \tan^{-1}\frac{-S\sin\omega}{(1-S)+S\cos\omega} \qquad (6)$$

S will affect the pitch and the upper harmonics will be slightly sharpened or flattened relative to the fundamental. Introducing an all-pass filter into the delay line can ameliorate this problem. The phase delay of this new filter can be altered to compensate for unwanted phase delay effects from the first filter.

It would be possible to control loudness simply by multiplying the output of this system by a gain. A more realistic string sound can be achieved, however, by using the fact that softly plucked notes generally contain less high-frequency energy than notes that are strongly plucked. Applying a low-pass filter to the input set of random samples could thus simulate a softly plucked note.

Finally, to avoid clicks and to make the beginning and ending of notes sound more realistic, an amplitude envelope could be applied to each note. Alternatively, the loss factor could be increased with time to make the note die off.

Although the Karplus-Strong approach is efficient and does provide realistic sounding plucked string tones, it does not, at first glance, appear to model any physical processes except in a very high-level sense. If this were true, then incorporating the effects of performance gestures such as plucking position or plucking material would involve somewhat artificial filtering techniques. It would be more useful to use techniques that truly attempt to model the physical processes of guitars, as this would perhaps make it more natural to incorporate control parameters.

It has been shown (Jaffe & Smith, 1983; Smith, 1993), however, that a waveguide implementation such as that shown in Figure 2 is in fact equivalent to a Karplus-Strong implementation of the type shown in Figure 1. Since this waveguide model is based directly on the simulation of physical processes, this means that the Karplus-Strong algorithm is more useful than it first appeared. In order to understand this, and to eventually consider superior systems to that shown in Figure 2, let us examine some of the physics behind string vibrations.

A SIMPLE STRING MODEL

The classic equation for an ideal string is:

$$\mu\frac{\partial^2 y}{\partial t^2} = F_x\frac{\partial^2 y}{\partial x^2} \qquad (7)$$

where μ is the mass per unit length of the string and F_x is the string tension. A traveling wave solution, known as the D'Alembert solution, is (Cuzzucoli & Lombardo, 1999):

$$y(x,t) = y_+(x-ct) + y_-(x+ct) \qquad (8)$$

where $y_+(x\text{-}ct)$ and $y_-(x+ct)$ are waves traveling in opposite directions with velocity *c*. These waves are set by the initial conditions of the system. The nut and bridge of the guitar can be approximated as ideal reflectors, with the result that *y(0,t) = y(L,t) = 0,* where L is the length of the string.

Equation 8 can be implemented on a computer using a data structure that consists of two arrays, one for each traveling wave. The displacement of any point on the string can be found be shifting the contents of one array left and the other right at the sampling rate, and finding the sum of the elements with corresponding indices.

Of course, this is only a very rough approximation. Aside from errors due to the finite resolution of spatial and temporal sampling, the following five physical characteristics of real systems are ignored (based on Cuzzucoli and Lobardo, 1999):

- Damping due to internal friction in the string and air resistance
- Loss of further energy due to interactions with the resonator
- Sympathetic vibrations excited by other strings or external sound sources
- Non-rigid terminations at the nut and bridge
- Dispersion due to finite elasticity of the string

REFINEMENTS TO THE SIMPLE STRING MODEL

The movement of a simple damped string is given by:

$$\mu \frac{\partial^2 y}{\partial t^2} = F_x \frac{\partial^2 y}{\partial x^2} - r(\omega)\frac{\partial y}{\partial t} \qquad (9)$$

where $r(\omega)$ is the damping coefficient that increases with frequency. For small displacements of the string, a solution to this partial differential equation is (Cuzzucoli & Lombardo, 1999):

$$y(x,t) = e^{\frac{-rx}{2\mu c}} y_+(x-ct) + e^{\frac{-rx}{2\mu c}} y_-(x+ct) \qquad (10)$$

This shows how amplitude decreases exponentially with time. It should also be noted that the time constant decreases with frequency:

$$\tau(\omega) = \frac{2\mu}{r(\omega)} \qquad (11)$$

Smith (1992) has suggested the possibility of simulating a damped string with non-rigid terminations by applying a proper load to a termination. For example, if the nut is considered to be at $x=0$, the vertical component of the string tension is, for small angles (Cuzzucoli & Lombardo, 1999):

$$F(t) = F_x \frac{\partial y}{\partial t} \qquad (12)$$

where F_x is the component of the tension parallel to the string at the nut.

Knowing that the equation for *F(t)* in equation 12 is the same as the definition of the mechanical impedance, Z_n, of the applied load, the following equation is arrived at giving the wave reflected from the nut:

$$y_+ = y_- \frac{R_c - Z_n}{R_c + Z_n} \qquad (13)$$

where $R_c = F_x / c$ is the string's characteristic resistance. This equation can be modeled with a digital filter with transfer function *H(z)*:

$$Y_+(z) = H(z)Y_-(z) \qquad (14)$$

In order to more accurately simulate a guitar string, a filter must be chosen that ensures that amplitude decreases with time, that higher partials decay faster than lower partials and that dispersion is avoided by ensuring that the phase relation between the components of signal is constant[1] (Cuzzucoli & Lombardo, 1999). A second order symmetrical finite impulse response filter with the following transfer function can fulfill these requirements:

$$H(z) = a_0 + a_1 z^{-1} + a_0 z^{-2} \qquad (15)$$

This filter has a constant delay of one sampling period. The values for the parameters in equation 15 can be determined by setting frequency-magnitude points. Cuzzucoli and Lobardo (1999) have suggested empirically determining different values for the gain at zero frequency and the cutoff frequency for each of the notes on each of the six strings to arrive at a natural sounding synthesis. Although this approach can produce a realistic sound, it is of limited utility in this form. Its reliance on empirical data rather than an analytical solution makes it inappropriate for situations where pitch can vary continuously, such as when a

[1] Although dispersion is present in heavy strings used by some instruments, such as pianos, guitar strings are flexible enough that it can be neglected.

string is bent or a slide is used. In cases such as these, it is necessary to interpolate to arrive at an appropriate pitch, which can result in unrealistic timbres. Furthermore, this approach does not directly encapsulate parameters such as plucking position or plucking material.

Before going on to discuss how this approach can be improved to include more sophisticated performance parameters, it is appropriate to mention a few techniques that can improve general performance. First of all, the quality of the sound could be further improved by considering the horizontal and vertical motion of the string independently (Jaffe and Smith, 1983). If the two models are slightly mistuned, a slight beating will result that will make the sound more natural and less synthetic. Another advantage of this two-models-per-string approach is that it allows strings to be coupled in a way that avoids feedback (Karjalainen, Valimaki and Tolonen, 1998). A matrix of coupling coefficients can be used to determine the proportion of the signal from each string that is sent to the other strings to simulate sympathetic vibrations.

INCORPORATING FURTHER PARAMETERS

As discussed earlier, plucking position can have a significant effect on the timbre of a note. For example, notes plucked close to the bridge tend to have richer high-frequency components then notes plucked closer to the centre of the string. Furthermore, plucking at a spot that corresponds to an anti-node of some harmonics of a string will emphasize those harmonics. Similarly, placing a finger at a potential node will also affect the spectrum.

An ideal string with length l and fixed ends can be described as a sum of modes (Traube & Smith, 2000):

$$\sum_n (A_n \sin \omega_n t + B_n \cos \omega_n t) \sin(k_n x) \tag{16}$$

where

$$A_n = \frac{2}{\omega_n l} \int_0^l y'(x,0) \sin(\frac{n\pi x}{l}) dx \tag{17}$$

and

$$B_n = \frac{2}{l} \int_0^l y(x,0) \sin(\frac{n\pi x}{l}) dx \tag{18}$$

The amplitude of the nth mode is thus

$$C_n = \sqrt{A_n^2 + B_n^2} \tag{19}$$

If the plucking action is idealized as a deformation of the string into a triangle with its summit at the point *(p,h),* where p is the distance from the end of the string and h is the height of deformation (amplitude), and if the string is considered to have no initial velocity, then the amplitude of the nth mode can expressed as:

$$C_n = \frac{2h}{n^2 \pi^2 R(1-R)} \sin(n\pi R) \tag{20}$$

where $R = p / l$.

A rudimentary approach to incorporating plucking position into synthesis would be to use additive synthesis to add the appropriate modes together. Of course, this ignores issues such as plucking material and duration of the pluck, and assumes a number of unrealistic conditions.

Karjalainen, Valimaki and Tolonen (1998) have suggested a fairly simple approach to simulating picking position. They argue that an adjustable comb filter could be used to simulate the plucking position. The delay would correspond to the time needed for the excitation to travel to the nut and back. This would result in a series of zeroes in the transfer function at frequencies $f_m = m / t_D$, where t_D is the delay of the comb filter and m is a non-negative integer. They also suggest using a low-pass filter with a cutoff frequency below the lowest fundamental frequency to be synthesized to approximate the output of energy at the bridge.

Cuzzucoli and Lombardo (1999) have suggested a more sophisticated, but still somewhat simplified method for modelling the parameters of a performer's touch. A plucker can be defined as a physical body with a mass M_d, a stiffness K_d, and a damping coefficient R_d. The following equation can then be used to relate the force acting on the string, *F(t),* to the external force applied by the plucker:[2]

$$F(t) = F_0(t) - (M_d + \mu\Delta)\frac{\partial^2 y}{\partial t^2} - R_d \frac{\partial y}{\partial t} - K_d y \tag{21}$$

If the string is plucked at some plucking point, x_p, four traveling waves will result: the wave traveling towards the bridge, the wave traveling towards the nut and the

[2] This equation assumes linear compression and a simple fluid damping coefficient.

two waves resulting from the reflections of these waves. The resulting string displacement will be:

$$\begin{aligned} y(x,t) &= f(x-ct)+g(x+ct)+h(x+ct) \quad \text{for } x<x_p \\ y(x,t) &= f(x-ct)+g(x+ct)+h(x-ct) \quad \text{for } x>x_p \\ y(x_p,t) &= f(x_p-ct)+g(x_p+ct)+h(x_p,t) \quad \text{for } x=x_p \end{aligned} \tag{22}$$

The functions f(x,t) and g(x,t) are related to the impedances of the terminations and the excitation wave h(x,t) is related to F(t), the force applied at x_p.

Equation 21 can be represented in the discrete domain using the following equation, at the sampling time m and at the plucking point:

$$\begin{aligned} F(m) &= F_0(m)\frac{M_d+\mu\Delta}{T^2}\left[y(n_p,m+1)+y(n_p,m-1)-2y(n_p,m)\right] \\ &-\frac{R_d}{2T}\left[y(n_p,m+1)-y(n_p,m-1)\right]-K_d y(n_p,m) \end{aligned} \tag{23}$$

This leads, after some math, to the excitation values at the plucking point:

$$\begin{aligned} h(m+1)c_0 &= h(m-1)c_1+\left[h(m)+w(m)\right]c_2+ \\ & w(m-1)c_3+w(m+1)c_4+F_0c_5 \end{aligned} \tag{24}$$

where w(n,m) = f(n,m) + g(n,m). Cuzzucoli and Lombardo went on to experimentally determine plausible values for certain performer actions and types of pluckers. Emphasis was put on considering not only the magnitude of the plucking force, but its duration and evolution with time as well. They found that the plucker's parameters had little effect on the sound when the force increased linearly with time during plucking, but that the parameters did have an influence if the pluck was a quick steady motion. This is because it did not matter how the string came to its final displacement in the first case. In addition, higher partials tended to prevail when the string was released rapidly, whereas they were somewhat suppressed when the string was released gradually. The properties of the plucking material related to friction have an effect here as well as the speed at which the string is released. All of these properties can be modeled by altering the equation for F(t).

The muting of strings is another important aspect that must be considered. Touching a string without exerting any appreciable force can mute a string. The effect on a string that is already sounding can be simulated by appropriately modifying the damping coefficient. In the case of plucking a muted string, the properties of the nut terminator need to be replaced by the properties of the damping finger. If a string that is already in motion is plucked, the plucking motion will damp the previous excitation. Equation 24 can be modified to reflect damping (Cuzzucoli and Lombardo, 1999):

$$h(m+1)=h(m)+\frac{\alpha}{1+\alpha}\left[w(m-1)-w(m+1)\right] \tag{25}$$

where

$$\alpha=\frac{RT}{2\mu\Delta} \tag{26}$$

and R is the damping coefficient.

THE RESONATOR

The bridge of a guitar transfers some of the energy from the vibrating strings to the body of the guitar. The body then vibrates itself and, in turn, transfers energy to the surrounding air as sound. Each guitar body has its own set of modes. It is very difficult to fully deal with the oscillatory behaviour of a guitar over the entire audio range, as the interaction between the resonant peaks of the strings and of the guitar body are difficult to model, particularly at higher frequencies (Cuzzucoli & Lombardo, 1999).

One approach to modelling the behaviour of the body is to use filters. Unfortunately, this can be very computationally intensive. Not only are there large number of resonances from 100 Hz up, but the temporal envelope of the body's response can be very complex (Karjalainen, Valimaki & Janosy, 1993). In order to solve this problem, Karjalainen, Valimaki and Janosy suggest the use of a chain of linear sub-systems:

$$y(n)=e(en)*s(n)*b(n) \tag{27}$$

where *e(n)* is the excitation source (i.e. a plucked string), *s(n)* is the impulse response of the string and *b(n)* is the impulse response of the body. Note that * is the convolution operator. The impulse response of the body can be pre-measured or pre-computed and stored in wavetable that can be accessed for each excitation event. This simplifies the amount of calculation that needs to be performed.

A simplified model of the guitar represents the resonator as a simple enclosure with a hole in it, which is to say a Helmholtz resonator. The motion of the strings excite the plate resonances of the guitar, which in turn excites the air within the guitar body as well as the external air. This results in an external force, due to air pressure, on the front and back plates of the guitar. In addition, there is a feedback relation due to sympathetic vibrations set up in the strings of the guitar. The acoustics of the room also play a role. All of this makes the guitar resonator very difficult to model beyond the first few modes of vibration. This makes simulations of the resonator with filters very difficult. Even if a great deal of computational power is available, it will not be helpful if parameters for the filters are not always available. Although it has been suggested that the higher frequency content could be simulated with noise, this is a low fidelity approach.

One practical solution to this problem (Karjalainen, Valimaki and Janosy, 1993; Karjalainen, Valimaki and Tolonen, 1998) is to simulate the behaviour of the resonator by appropriately modifying the excitation signal. Since the convolution operator is commutative, equation 27 can be modified to:

$$y(n) = b(n) * e(n) * s(n) \tag{28}$$

A recording of an actual guitar note could then be used as the excitation source, since it would include the behaviour of the guitar body. Unfortunately, this approach limits the available parametric control, since the exciter is a pre-recorded signal. Karjalainen and Smith (1996) have suggested techniques for combining this approach with a body filter approach to achieve more control while still achieving efficient synthesis. While this could be useful in the short term, it does not truly model the physics behind guitar bodies. This approach is therefore not very useful if one desires to be able to model an arbitrary body including, perhaps, bodies that might be impossible or difficult to build. It is clear that, while some progress has been made, there is still a need for significant improvements before a system can be said to truly model guitar bodies in a realistic, flexible and easily controllable way.

CURRENT STATE OF THE ART

Laurson et al. (2001) have produced the most recent published system. Their basic system is based on commuted waveguide synthesis and is composed of a feedback loop that contains a delay line and two digital filters. They chose to use an FIR filter instead of an allpass filter to compensate for fractional delay because they found it produced better glissando and cleaner vibrato. Two such basic systems were used for each guitar string, since the x and y dimensions were modelled separately.

Plucking point was simulated using a comb-filter and a recording of an actual guitar signal was used as the excitation signal. A database of excitation signals indexed based on fret position and string was used. The horizontal dimension of each string was coupled with that of the other strings in order to simulate sympathetic vibrations. Finally, a database of special effects, such as scraping of the string or knocking on the guitar body, was available to be added to the output signal. Rules were also implemented to generate envelopes for performer parameters.

Although this system did not offer anything entirely new in terms of pure synthesis, it did combine a number of ideas into a functioning and practical system that was reportedly successful. However, it did not accurately model many of the performance gestures discussed in the introduction. Furthermore, because it relies on a database of pre-recorded information rather than true mathematical modelling, this system is somewhat limited. Nonetheless, this system can likely be considered the state of the art in terms of realistic sounding synthesis.

CONCLUSIONS

A number of successful techniques have been devised and implemented to simulate realistic sounding simple plucked strings. Some initial work has also been done on simulating plucking parameters as well as muting of strings and sympathetic vibration. Although plucking position can be modelled fairly well, the effects of different plucking materials and picking direction have only been considered on a fairly rudimentary level. The physical parameters of the strings, nut and bridge have also been modelled in a more or less idealized manner. There is also a great deal of work that needs to be done before guitar resonators and their interactions with the strings and the surrounding environment can be simulated efficiently and effectively.

Aside from some early rudimentary work (Jaffe and Smith, 1983), very little has been done to effectively simulate left-hand techniques such as bends, hammer-ons, pull-offs, trills, fret tapping and slides. Furthermore, the reliance on recorded excitation signals limits the generality of systems and does not take into account excitation from the left hand (e.g. hammer-ons) as opposed to the right hand. It also makes it difficult to deal with situations where a guitar is played with a slide, since string length is varied continuously and it is not possible to record excitation signals for all possible slide positions.

So, although the groundwork has certainly been laid for guitar synthesis, there is still much to be done before a synthesizer is arrived at that can realistically synthesize the full range of common performance gestures. The work that has been done, however, shows that it is certainly possible to arrive at a reasonably high quality synthesis using simplifications. This sets a baseline that can be expanded upon in future research.

REFERENCES

Cuzzucoli, G. and V. Lomabrdo. 1999. "A Physical Model of the Classical Guitar, Including the Player's Touch." *Computer Music Journal*. 23(2): 52-69.

Jaffe, D. A., and J. O. Smith. 1983. "Extensions of the Karplus-Strong Plucked-String Algorithm." *Computer Music Journal*. 7(2): 76-87.

Karjalainen, M., V. Valimaki and Z. Janosy. 1993. "Towards High-Quality Sound Synthesis of the Guitar and String Instruments." *Proceedings of the International Computer Music Conference*. 56-63.

Karjalainen, M. and J. Smith. 1996. "Body Modelling Techniques for String Instrument Synthesis." International Computer Music Conference.

Karjalainen, M., V. Valimaki and T. Tolonen. 1998. "Plucked-String Models: From the Karplus-Strong Algorithm to Digital Waveguides and Beyond." *Computer Music Journal*. 22(3): 17-32.

Laurson, M. et al. 2001. "Methods for Modelling Realistic Playing of Acourstic Guitar Synthesis." *Computer Music Journal*. 25(3): 38-49.

Moore, R. F. 1990. *Elements of Computer Music*. Upper Saddle River, N.J.: Prentice Hall.

Roads, C. 1996. *The Computer Music Tutorial*. Cambridge: MIT Press.

Smith, J. O. 1992. "Physical Modelling Using Digital Waveguides." *Computer Music Journal*. 16(4): 74-91.

Smith, J. O. 1993. "Efficient Synthesis of Stringed Musical Instruments." *Proceedings of the International Computer Music Conference*. 64-71.

Tolonen, T. and V. Valimaki. 1997. "Automated Parameter Extraction for Plucked String Synthesis." *Proceedings of the International Symposium on Musical Acoustics*. 245-250.

Traube, C. and J. O. Smith. 2000. "Estimating the plucking point on a guitar string." *Proceedings of the COST G-6 Conference on Digital Audio Effects*. 153-8.