Automatic Chord Labelling: A Figured Bass Approach

Yaolong Ju^{1,2}, Sylvain Margot^{1,2}, Cory McKay^{2,3}, Ichiro Fujinaga^{1,2}

² Centre for Interdisciplinary Research in Music Media and Technology (CIRMMT), Canada

³ Department of Liberal and Creative Arts, Marianopolis College, Canada

 $\{yaolong.ju, sylvain.margot, cory.mckay\} @mail.mcgill.ca, ichiro.fujinaga@mcgill.ca, ichiro.fujinaga$

ABSTRACT

Automatic chord labelling is challenging, largely because the identification of chords directly from the musical surface can be ambiguous. Figured bass can potentially offer indications of harmonic rhythm and non-chord tones, thereby reducing this ambiguity. This paper proposes a series of four rule-based algorithms that automatically generate chord labels for homorhythmic Baroque chorales based on both figured bass annotations and the musical surface. These are applied to the existing Bach Chorales Figured Bass dataset, which consists of 139 chorales composed by Johann Sebastian Bach, and includes both the original music and figured bass annotations. Analysis of the chord labels produced by our algorithms reveals occasional discrepancies between the chords implied by the figured bass and the scored voices, something that provides a useful basis for exploring different chord interpretations. The chord annotations produced by our system are presented as the new Bach Chorales Multiple Chord Labels (BCMCL) dataset, which provides a choice of four parallel chord labels for each chorale. These range from one set of labels based only on the figured bass, which do not assume any music theoretical ideas proposed after the time the chorales were written, to a set of labels based on both the figured bass annotations and the full musical surface that considers the music from the perspective of modern tonal music theory. It is hoped that this dataset and the algorithms used to label it will be of interest for both future musicological research and research on automatic chord labelling systems.

CCS CONCEPTS

• Information systems → Music retrieval; • Applied computing → Sound and music computing; • Computing methodologies → Rule-based approaches.

KEYWORDS

figured bass, chord labels, harmonic ambiguity, heuristics

ACM Reference Format:

Yaolong Ju^{1,2}, Sylvain Margot^{1,2}, Cory McKay^{2,3}, Ichiro Fujinaga^{1,2}. 2020. Automatic Chord Labelling: A Figured Bass Approach. In *7th International Conference on Digital Libraries for Musicology (DLfM 2020), October 16,* 2020, Montréal, QC, Canada. ACM, New York, NY, USA, 5 pages. https: //doi.org/10.1145/3424911.3425513

© 2020 Association for Computing Machinery.

ACM ISBN 978-1-4503-8760-6/20/10...\$15.00

1 INTRODUCTION

In general, chord labelling refers to the identification of chords from the musical surface.¹ Many prominent music theorists (e.g., Rameau, Weber, Riemann, and Schoenberg) have proposed different approaches to chord labeling, and recent studies show that analysts often disagree with one another, and are sometimes not even internally consistent [6, 8, 13, 15, 16, 20]. Given the subjectivity and inconsistency of chord labelling, it is challenging to automate it.

Figured bass annotations (FBAs), a type of music notation commonly used in Baroque music that uses numerals and other symbols to indicate intervals above the bass line (or "continuo"), are considered one of the earliest ways to imply chord-like labels [2, 22]. As shown in Fig. 1, the figures in the score (e.g., "5/3" at m. 1.2)² imply accompanying chords to be improvised by musicians such as keyboard or lute players. There are three aspects of figured bass notation that should be highlighted:

- Backslashes through numbers usually indicate raised intervals (e.g. mm. 2.2.5³, and 5.2 of Fig. 1) and forward slashes indicate lowered intervals.
- (2) Continuation lines (e.g., m. 1.3.5 of Fig. 1) indicate that the chord of the preceding figure is prolonged.
- (3) Multiple FBAs below a stationary bass note (e.g., 9–8 in m. 4.2 of Fig. 1) may⁴ indicate a suspension being resolved.

Although the study of figured bass has been a standard topic in music theory, music pedagogy, and musicology, automated approaches to figured bass processing have only drawn limited attention [4, 14, 21]. In this paper, we propose an innovative, rulebased model for automatic chord labelling that considers both the musical surface and figured bass. Compared to existing methods that only considered the musical surface [5, 6, 9, 13, 15, 18, 19], the advantages of our approach are:

¹ Schulich School of Music, McGill University, Canada

Permission to make digital or hard copies of all or part of this work for personal or classroom use is granted without fee provided that copies are not made or distributed for profit or commercial advantage and that copies bear this notice and the full citation on the first page. Copyrights for components of this work owned by others than ACM must be honored. Abstracting with credit is permitted. To copy otherwise, or republish, to post on servers or to redistribute to lists, requires prior specific permission and/or a fee. Request permissions from permissions@acm.org. *DLfM 2020, October 16, 2020, Montréal, QC, Canada*

https://doi.org/10.1145/3424911.3425513

 $^{^1\}mathrm{The}$ musical surface can be understood as the specific notes indicated on a score. Figured bass annotations are not considered part of the surface.

²The "5/3" in the first measure indicates pitch classes "C" and "A", which respectively form the 5th and 3rd intervals above the bass "F", and thereby imply an F major chord. m. 1.2 means the second beat of the first measure. Pickup measures are excluded from this count.

³ "m. 2.2.5" means the second measure, the second and half beat.

⁴To confirm a suspension, one must examine the musical surface to see whether the suspension is prepared properly.



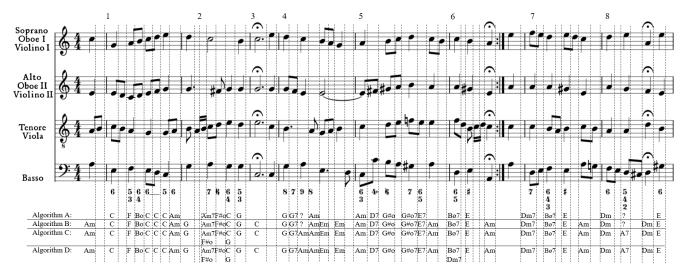


Figure 1: The first measures of BWV 33.06 "Allein zu dir, Herr Jesu Christ" from our Bach Chorale Figured Bass (BCFB) dataset. FBAs and chord labels are shown below the bass line. The vertical dashed lines divide the music into a series of note onset slices, which are formed whenever a new note onset occurs in any voice; each slice consists of the vertical set of pitch classes sounding at that moment. The results produced by each of our four chord labelling algorithms (see Section 2) are indicated below the music, separated by horizontal lines.

- Figured bass offers some indications of harmonic rhythm,⁵ non-chord tones,⁶ [10, 11], thereby potentially reducing the level of harmonic ambiguity. Fig. 2 provides an example of how it can be ambiguous to label certain chords without FBAs: two different theorists provided chord labels based only on the musical surface, resulting in disagreement at six spots in just two measures of music. Fortunately, FBAs may help to resolve this ambiguity. For example, in m. 5.2.5 the FBA indicates a chord change,⁷ which suggests the passing D should be interpreted as a chord tone, thus forming a new "Em7" chord. In the cases of m. 5.3.5 and m. 5.4.5, both spots are left unfigured, suggesting no chord change, which means that the passing notes can be interpreted as non-chord tones, so the chord labels remain unchanged from the preceding slices ("F" and "Bo", respectively).
- Since FBAs are often attributed directly to composers, as opposed to copyists or editors, the chord labels they imply may offer meaningful insights into a composer's unique compositional style.⁸ This applies especially to intentions relating to counterpoint and harmony, which are of interest both to the generation of authoritative ground truth for automated systems and to music theoretical and musicological study.

In this paper, we apply our automatic chord labelling model to the Bach Chorale Figured Bass (BCFB) dataset [14], a corpus we constructed containing FBAs in MusicXML, **kern, and MEI (Music Encoding Initiative) formats.⁹ We chose this repertoire due to its key role in modern music pedagogy and its general historical importance. It consists of all 139 Johann Sebastian Bach chorales that include figured bass he wrote himself, based on the Neue Bach Ausgabe (NBA) [3], the most up-to-date scholarly critical edition.



Figure 2: Measures 5 and 6 of BWV 248.05 "Klagt, Kinder, klagt es aller Welt". Each of the two rows of chord labels was annotated manually by a different music theorist based only on the musical surface. One can see that their annotations did not always agree, which suggests a degree of harmonic ambiguity. Figured bass can help resolve such disagreements. Chord labels supported by the figured bass are marked in red.

2 METHODOLOGY

We propose a series of rules that can be applied to generate chord labels from both FBAs and the musical surface. These rules are based on treatises that discuss figured bass and chords [1, 2], and on consultation with expert music theorists. For ease of

⁵Harmonic rhythm means the rate at which chords change in the music.

⁶Non-chord tones that are part of suspensions are indicated in the FBAs by voiceleading motions (e.g., 9–8). Other types of NCTs, such as passing tones or neighbor tones, are implied by the absence of the corresponding notes in FBAs.

 $^{^7\}mathrm{In}$ this paper, we consider that each slice with figures represents an individual chord.

⁸Although figured bass is primarily a notation for performance, rather than a strict prescription of harmony, it nonetheless provides a useful description of harmony acknowledged by others [2, 7].

⁹Available at: https://github.com/juyaolongpaul/Bach_chorale_FB.

DLfM 2020, October 16, 2020, Montréal, OC, Canada

understanding and to permit empirical comparisons, we have divided the associated processing steps into four algorithms (A, B, C, and D), each of which successively incorporates all of the processing in the previous algorithm and adds additional new rules.

As an initial pre-processing step, the music is segmented into a series of note onset slices [12, 17], which are formed whenever a new note onset occurs in any voice; each slice consists of the vertical set of pitch classes sounding at that moment, as shown in Fig. 1. Then, for each slice, each of the four algorithms outputs one or more root pitch classes and one or more of nine candidate chord qualities: major, minor, diminished, and augmented triads, as well as major, minor, dominant, half-diminished, and fullydiminished seventh chords.¹⁰ Fig. 1 demonstrates sample output for each of the algorithms.

Algorithm A: This is the baseline algorithm, which generates chord labels based only on the bass notes and the FBAs (unlike algorithms B, C, and D, which also consider all voices of the musical surface). For each figured slice, Algorithm A will consider all the pitch classes (PCs) implied¹¹ by the FBA as chord tones above the bass note.¹² If the quality of the chord has one of the nine candidate qualities specified above, then the slice is labeled with this chord. Otherwise, the slice will be labelled with "?". There is also additional logic added to the processing in order to account for rare cases, based on the contemporary rules by Johann Heinichen and George Telemann, summarized in Arnold (1931, 263, 311).¹³ The output of Algorithm A is always a single chord label for each slice with a figure, and no label for slices without figures.

Algorithm B: It was common shorthand for Baroque composers to omit figures corresponding to root position major or minor triads (e.g., mm. 2.1, 3.1, and 4.3 of Fig. 1). This, of course, does not imply that such slices are not harmonically important or that accompanists should ignore them. Algorithm B, a superset of Algorithm A, therefore, introduces new processing that considers the full musical surface (as well as the FBAs) to label all unfigured slices consisting of root position triads. Overall, Algorithms A and B focus on converting FBAs to chord labels according to 18th-century treatises, without imposing modern theories of chord labeling (e.g., suspensions or non-chord tones in the bass).

Algorithm C: Departing from Algorithm A and Algorithm B, Algorithm C begins to incorporate modern approaches to chord labelling. Suspensions are an indispensable part of modern chord labelling practice, so Algorithm C (a superset of Algorithm B) identifies suspensions based on both the FBAs and the musical surface. There can, however, be multiple ways of legitimately labelling suspensions with chords. For example, in the case of "7-6" suspensions, "6-5" suspensions, and cadential "6/4" suspensions (established by either "4-3" or "6/4-5/3" suspensions), suspended notes can be either treated as chord tones, resulting in, respectively, a seventh chord (e.g., "Am7" at m. 2.2 of Fig. 1), a

sixth chord, or a 6/4 chord (e.g., "C" at m. 2.3 of Fig. 1), or they can be treated as non-chord tones, in which case they should adopt the chord label from the slice to which the suspension is resolved (e.g., "F\$o" at m. 2.2 and "G" at m. 2.3 of Fig. 1). Other types of suspensions can be dealt with similarly. Algorithm C outputs two possible chord labels in such cases, in order to reflect both theoretically viable options. Other slices not involving such suspensions are each annotated with a single chord label, according to Algorithm B. Additionally, if a slice is figured (e.g., the "5/4/2" slice at m. 8.2 of Fig. 1) but the chord quality is not one of the recognized types, then the algorithm adopts the chord label from the subsequent¹⁴ slice (e.g., the "A7" chord label¹⁵ at m. 8.2 of Fig. 1).

Algorithm D: It can happen that a PC explicitly specified by the figured bass is not found in the musical surface, such as in m. 6.1 of Fig. 1, where the bass is "D" and the figured bass suggests a "6/5" chord ("Bø7"), but the musical surface suggests a seventh chord ("Dm7"), because the "B" explicitly specified by the figure "6" is not in the surface. Such discrepancies are valuable to track, not only because they identify two possible chord labels, but also because they demonstrate a situation where the figured bass and the musical surface appear to disagree, which may provide new insights to musicology and music theory. We therefore implemented Algorithm D, which is a superset of Algorithm C, that identifies such discrepancies and adds the corresponding alternative chord label for slices where the figured bass and musical surface disagree. Note that Algorithm D does not track the reverse case; that is, no alternate chord label is generated when a pitch in the surface is not indicated in the figure (e.g., in m 1.4.5 of Fig. 1, the "G" that would correspond to a 7th of "A" is in the surface, but is not indicated in the figure), since we consider pitches excluded by the figures as non-chord tones.

The open source code implementing each of these algorithms can be found at: https://github.com/juyaolongpaul/harmonic_ analysis/.

3 **BACH CHORALES MULTIPLE CHORD** LABELS DATASET

Here, we introduce the new Bach Chorales Multiple Chord Labels (BCMCL) dataset, which includes 120¹⁶ of the 139 BCFB chorales, now annotated with the chord labels output by each of our four algorithms described in Section 2. The parallel tracks of chord labels produced by our four algorithms allow users to access the kinds of labels most relevant to their own research; for example, those conducting historical research may be interested in the Algorithm A or Algorithm B labels, which do not impose any non-contemporary harmonic models, while those working on training automatic chord classifiers based on modern harmonic analysis may be more interested in the Algorithm D labels. Comparing the differences between the label tracks may also be of musicological or music theoretical interest. The BCMCL music and annotations are freely available at: https: //github.com/juyaolongpaul/BCMCL.

 $^{^{10}\}mathrm{We}$ limit the output to just these qualities for the sake of simplicity.

¹¹Take m. 5.3 of Fig. 1 as an example. Here the figure is "7", which indicates a root position seventh chord above the bass "G#." We know that by definition a seventh chord consists of intervals of a 3rd, 5th, and 7th above the root, so next we can look at the key signature to find the diatonic pitch class set, which in this case is [C, D, E, F, G, A, B]. Therefore, a 3rd, 5th, and 7th above the bass respectively indicate pitch classes of "B", "D", "F" pitch classes, which along with the bass "G#" represent a "G#07" chord. ¹² See the 18th-century figured bass treatise by Carl Philipp Emanuel Bach [2] for

the foundations of this methodology.

¹³For example, m. 5.1.5 of Fig. 1 the "4+" also implies "6" and "2" over the bass, forming a "D7" chord.

 $^{^{14}\}mathrm{In}$ such cases, the chord is labelled based on the musical surface only.

¹⁵This aligns with the figured bass treatise by Heinichen (Arnold 1931, 261), where a "5/4/2" figure is considered as the first inversion of a section of "5/4/2" figure is considered as "the first inversion of a Seventh with retarded bass." phrases (BWV 24.06, 76.07, 100.06, 105.06, 113.01, 129.05, 167.05, 171.06, 248.09, 248.23, 248.42, and 248.64) from BCFB because they deviate significantly from the homorhythmic texture for which our automatic chord labelling model is built. We also excluded five other chorales (BWV 16.06, 48.07, 149.07, 195.06, and 447) that are barely figured. Finally, we excluded BWV 8.06 and BWV 161.06 because they feature irregular textures, such as having an obbligato continuo and/or instrumental part.

DLfM 2020, October 16, 2020, Montréal, QC, Canada

Category	Number
Chorales	120
Note onset slices	9,617
Candidate chord qualities	9
Chord types	109
Chord labels (from Algorithm D)	10,092
Slices with suspensions	312 (3.24%)
Slices with two chord interpretations	471 (4.90%)
Slices with FB / surface discrepancies	276 (2.87%)

Table 1: The number of chorales, note onset slices, candidate chord qualities, chord types (identified by the combination of chord root and quality), and chord labels (including all labels for all slices produced by Algorithm D) in the BCMCL dataset. Total slice counts and percentages (divided by the number of note onset slices) are also provided for slices with suspensions (resolutions not included), slices with two legitimate chord labels, and discrepancies between the figured bass and musical surface.

Table 1 summarizes certain statistics on BCMCL, based on the chord labels produced by Algorithm D.¹⁷ The distributions of chord types and qualities are shown in Fig. 3. Note that the top 20 chord types represent 75.1% of all chords, and these are mostly major or minor triads. Seventh chords are less common, with their most frequent qualities being dominant seventh (7.5%) and minor seventh (5.8%). Augmented triads, major seventh, halfdiminished, and fully diminished seventh chords are relatively rare in BCMCL.

We can observe from the distributions of suspensions (left of Fig. 4) that the majority of suspensions are of the "4–3" type, followed by "7–6", "9–8", and "6–5" suspensions which are less frequent, but still happen regularly. Double suspensions ("6/4–5/3" and "9/4–8/3") exist but are rare in BCMCL. Also, looking at the discrepancies between the figured bass and the surface (right of Fig. 4), the pitch classes indicated by the figures "6" and "5" are particularly likely to be absent in the musical surface.

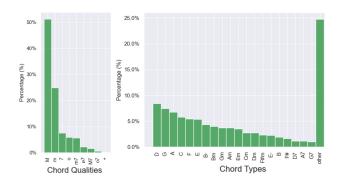


Figure 3: Distributions of chord qualities (left) and chord types (right) for all 10,092 chords in BCMCL.

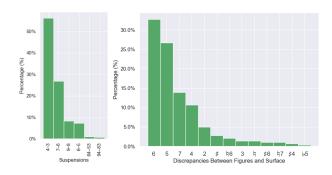


Figure 4: Distributions of suspensions (left), and discrepancies between the figured bass and surface (right). In the latter graph, each column corresponds to an interval found in the figured bass but absent in the surface. The figure "3" is sometimes omitted from the figured bass: in such cases, "#" and "#" mean raised third and natural third, respectively.

4 CONCLUSION AND FUTURE RESEARCH

We proposed four rule-based algorithms that generate chord labels automatically based on both figured bass annotations and the musical surface. Figured bass can serve as a guide for harmonic rhythm and non-chord tones. We applied our system to Bach chorales, and present the resulting parallel chord annotations as the Bach Chorales Multiple Chord Labels (BCMCL) dataset, which we hope will facilitate future research revealing insights into Bach's unique compositional style, especially plausible possible ideas about his thoughts on counterpoint and harmony. The four separate tracks of chord labels may provide an interesting comparative resource. Statistics we calculated on BCMCL may also be of interest to musicological research on Bach's chorales.

Furthermore, BCMCL may be useful in automated chord classification research. By offering multiple chord labels per slice, when appropriate, it facilitates multi-label classification, an understudied but essential aspect of music information retrieval research. Even for systems that can only output single classifications per slice, datasets such as this, which specify multiple chords per slice, offer important advantages with respect to fairer evaluation of algorithms.

There are intriguing directions for future research. One is to more deeply study the context of the discrepancies between the figured bass and the musical surface; such studies may yield potential insight on how and why Bach figured his chorales. Also, our algorithm currently works only with homorhythmic music, so expanding it to work on music with other textures, especially homophony, would certainly be valuable. To facilitate the usage of figured bass and chord annotations, we aim to create a search engine and interface that would make them both searchable via a single interface.

ACKNOWLEDGMENTS

We would like to thank the Social Sciences and Humanities Research Council of Canada (SSHRC) and the Fonds de recherche du Québec-Société et culture (FRQSC) for their generous funding. We would also like to acknowledge the contributions of our many collaborators on the Single Interface for Music Score Searching and Analysis (SIMSSA) project, especially Julie Cumming and Samuel Howes.

¹⁷We chose to show the statistics of Algorithm D because they demonstrate useful information on the number of slices with two chord interpretations, and on the number of slices with figured bass (FB) / surface discrepancies. Slices left unlabelled by Algorithm D (e.g., m. 3.4 of Fig. 1) are assigned label(s) from the previous slice (e.g., "C" will be the chord label for m. 3.4) for the purposes of Table 1, Fig. 3, and Fig. 4.

Automatic Chord Labelling: A Figured Bass Approach

DLfM 2020, October 16, 2020, Montréal, QC, Canada

REFERENCES

- Franck Thomas Arnold. 1931. The Art of Accompaniment from a Thoroughbass: As Practised in the XVIIIth & XVIIIth Centuries. Oxford University Press, London, UK.
- [2] Carl Philipp Emanuel Bach. 1949. Essay on the True Art of Playing Keyboard Instruments. 2 vols. (Berlin, 1753 and 1762). Translated by W. J. Mitchell. W. W. Norton, New York, NY.
- [3] Johann Sebastian Bach, Alfred Dürr, and Werner Neumann. 1954–2007. Neue Ausgabe sämtlicher Werke. Bärenreiter, Kassel, Germany.
- [4] Jerome Barthélemy and Alain Bonardi. 2001. Figured Bass and Tonality Recognition. In Proceedings of the 2nd International Society for Music Information Retrieval Conference. Indiana, USA, 129–136.
- [5] Tsung-Ping Chen and Li Su. 2019. Harmony Transformer: Incorporating Chord Segmentation into Harmony Recognition. In Proceedings of the 20th International Society for Music Information Retrieval Conference. Delft, Netherlands, 259–267.
- [6] Nathaniel Condit-Schultz, Yaolong Ju, and Ichiro Fujinaga. 2018. A Flexible Approach to Automated Harmonic Analysis: Multiple Annotations of Chorales by Bach and Prætorius. In Proceedings of the 19th International Society for Music Information Retrieval Conference. Paris, France, 66–73.
 [7] Hans Theodore David, Arthur Mendel, and Christoph Wolff. 1999. The New
- [7] Hans Theodore David, Arthur Mendel, and Christoph Wolff. 1999. The New Bach Reader: A Life of Johann Sebastian Bach in Letters and Documents. W. W. Norton, New York, NY.
- [8] Trevor de Clercq and David Temperley. 2011. A Corpus Analysis of Rock Harmony. Popular Music 30, 1 (2011), 47–70.
- [9] Mark Thomas Granroth-Wilding. 2013. Harmonic Analysis of Music Using Combinatory Categorial Grammar. Ph.D. Dissertation. University of Edinburgh.
- [10] Ludwig Holtmeier. 2007. Heinichen, Rameau, and the Italian Thoroughbass Tradition: Concepts of Tonality and Chord in the Rule of the Octave. *Journal* of Music Theory 51, 1 (2007), 5–49. Duke University Press, Durham, NC.
- [11] Samantha M. Inman. 2018. Introduction to Graduate Theory: Teaching Tonal Hierarchy through Bach. Bach 49, 2 (2018), 345–364.
- [12] Yaolong Ju, Nathaniel Condit-Schultz, Claire Arthur, and Ichiro Fujinaga. 2017. Non-chord Tone Identification Using Deep Neural Networks. In Proceedings of the 4th International Workshop on Digital Libraries for Musicology. Shanghai, China, 1–4.

- [13] Yaolong Ju, Samuel Howes, Cory McKay, Nathaniel Condit-Schultz, Jorge Calvo-Zaragoza, and Ichiro Fujinaga. 2019. An Interactive Workflow for Generating Chord Labels for Homorhythmic Music in Symbolic Formats. In Proceedings of the 20th International Society for Music Information Retrieval Conference. Delft, Netherlands, 862–869.
- [14] Yaolong Ju, Sylvain Margot, Cory McKay, and Ichiro Fujinaga. 2020. Automatic Figured Bass Annotation Using the New Bach Chorales Figured Bass Dataset. In Proceedings of the 21th International Society for Music Information Retrieval Conference. To appear.
- [15] Hendrik Vincent Koops, W. Bas de Haas, Jeroen Bransen, and Anja Volk. 2020. Automatic Chord Label Personalization through Deep Learning of Shared Harmonic Interval Profiles. *Neural Computing and Applications* 32, 4 (2020), 929–939.
- [16] Hendrik Vincent Koops, W. Bas de Haas, John Ashley Burgoyne, Jeroen Bransen, Anna Kent-Muller, and Anja Volk. 2019. Annotator Subjectivity in Harmony Annotations of Popular Music. *Journal of New Music Research* 48 (2019), 1–21.
- [17] Pedro Kröger, Alexandre Passos, Marcos Sampaio, and Givaldo De Cidra. 2008. Rameau: A System for Automatic Harmonic Analysis. In Proceedings of International Computer Music Conference. Belfast, Ireland, 273–281.
- [18] Kristen Masada and Razvan Bunescu. 2019. Chord Recognition in Symbolic Music: A Segmental CRF Model, Segment-Level Features, and Comparative Evaluations on Classical and Popular Music. *Transactions of the International* Society for Music Information Retrieval 2, 1 (2019), 1–13.
- [19] Gianluca Micchi, Mark Gotham, and Mathieu Giraud. 2020. Not All Roads Lead to Rome: Pitch Representation and Model Architecture for Automatic Harmonic Analysis. *Transactions of the International Society for Music Information Retrieval* 3, 1 (2020), 42–54.
- [20] Yizhao Ni, Matt McVicar, Raul Santos-Rodriguez, and Tijl De Bie. 2013. Understanding Effects of Subjectivity in Measuring Chord Estimation Accuracy. *IEEE Transactions on Audio, Speech, and Language Processing* 21, 12 (2013), 2607–2615.
- [21] Adam Wead and Ian Knopke. 2007. A Computer-Based Implementation of Basso Continuo Rules for Figured Bass Realizations. In Proceedings of the International Computer Music Conference. Copenhagen, Denmark, 188–191.
- [22] Peter Williams and David Ledbetter. 2001. Figured Bass. Oxford Music Online (2001). Accessed March 4, 2020.