MUMT 618 Final Project Report: Differences Between Two Dimensional and Three Dimensional Acoustical Cylinders: A Study of Acoustic Properties Using Finite Element Method

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ABSTRACT

In order to save time in study of acoustic problems, people tend to use 2D geometry as a simplification of a fully 3D one. However, the simplification from 3D to 2D may bring in some errors. In this project, finite element method (FEM), implemented in COMSOL, is used to study the difference between 2D symmetrical (treated as 3D) and fully 2D models. A closed and an open cylinder are used for validating the method. Then both unflanged and flanged open pipe are studied for two different models. Input impedance, reflection coefficient and radiation impedance are calculated and compared between the theoretical results.

1. INTRODUCTION

The more and more powerful computational capabilities make it possible to directly study acoustic problems in three dimension (3D) domain. Even though, in order to same computing time, two dimension (2D) geometries are frequently used as a simplification for 3D problems, like voice production, instrument sound synthesis. However, this might bring errors in the final result.

In order to study differences between 2D and 3D models, two detailed studied geometries, open unflanged or flanged cylinder could be used. The radiation impedance Z_r and complex reflection coefficient R of an open cylinder is an important parameters to study the acoustic behaviors of the pipe. The resonance frequencies and capability of radiating sound could also be determined. Theoretical values for reflection coefficient and end correction length are available for both open end unflanged [1] and infinite flanged [2] cylinders . A further approximated formula for unflanged [3] and infinite flanged [4] cylinders are also developed latter. In order to study the varies geometries where the analytical solution does not exist. Numerical methods like boundary element method (BEM) and finite difference method (FDM) are used [5, 6].

Here in this project, FEM will be used to study the acoustic properties of unflanged and infinite flanged cylinders based on different models using commercial software COMSOL. The paper is structured as follows. Section 2 will introduce several basic acoustic concepts and theories. In section 3, simple cases will be calculated in order to validate the eligibility of COMSOL in this study. Section 4 compares several acoustic properties simulated by two different models for both unflanged and flanged open cylinders. Finally, the conclusions and future works are presented in section 5.

2. GENERALITIES

In these section, several basic acoustic concepts and theories will be introduced.

2.1. Acoustic impedance

Acoustic impedance, defined as $Z = p/U$, is usually used to depict the frequency properties of a certain geometry. p is the acoustic pressure and U is the volume flow velocity. It indicates how much sound pressure is generated by a given air vibration at a certain frequency.

Specific acoustic impedance Z_s depends on the property of the medium given by $Z_s = \rho c$.

Characteristic acoustic impedance Z_c also depends on the size of the pipe and given by $Z_c = \rho c/A$, where A is the cross-section area of the pipe.

Input impedance Z_{in} is the impedance at the driven point of the pipe where the air is excited.

Load impedance Z_L is the impedance set at the output end of the pipe. For an open pipe, $Z_L = 0$. For a closed pipe, Z_L is infinity.

Radiation impedance is related to the reflection coefficient R_r at the open end of the cylinder given by

$$Z_r = Z_c \frac{1 + R_r}{1 - R_r} \tag{1}$$

Theoretical input impedance could be calculated based on the load impedance Z_L by the following equation

$$Z_{in} = Z_c \frac{1 + C_-/C_+}{1 - C_-/C_+} \tag{2}$$

where C_-/C_+ could be represented by Z_L and Z_c as

$$C_-/C_+ = \frac{Z_L/Z_c - 1}{Z_L/Z_c + 1} e^{-2jkL} \tag{3}$$

2.2. Cut-off frequency

For wave propagating inside cylinder pipe, in addition to the longitudinal wave motion along the main axis, there are higher modes. But the higher mode is evanescent for frequencies less than the first cut-off frequency. So only the first mode, that means plane wave exists. For the first propagating mode, cut-off frequency is given by

$$f = \frac{1.84c}{2\pi a} \tag{4}$$

where $ka = 1.84$ is the Helmholtz number, c is the sound speed and a is the radius of the pipe. However, if the instrument had a radial symmetry, the first cut off frequency would be given by $ka = 3.83$ [7];

2.3. Thermoviscous losses

In practical problems, there will be losses along the wall due to the viscous drag and thermal losses. A special boundary condition will be used for thermoviscous boundary layer losses. As used by Harazi in previous research, wall admittance is introduced:

$$Y_s = \frac{1}{2}(1-j)\frac{\omega}{\rho c^2}[l_{vor} sin^2\theta_i + (\gamma - 1)l_{ent}] \tag{5}$$

where

$$l_{vor} = \frac{1}{|k_{vor}|} = \sqrt{\frac{2\mu}{\omega\rho}},$$
$$l_{ent} = \frac{1}{|k_{ent}|} = \sqrt{\frac{2\kappa}{\omega\rho C_p}} = \frac{l_{vor}}{\sqrt{Pr}},$$
$$Pr = \frac{\mu C_p}{\kappa},$$

where θ_i is the angle of incidence with the bore, k_{vor} and k_{ent} are the wave number of the vorticity and entropy modes, μ is the dynamic viscosity, C_p is the specific heat capacity, κ is the thermal conductivity, and Pr is the Prandtl number.

2.4. Reflection coefficient

For a plane wave inside a cylinder, it could be considered as the sum of the wave propagating in positive and negative directions. For a certain point along the axis x, the pressure is given by:

$$P(x) = p^+(\omega)e^{-jkx} + p^-(\omega)e^{jkx} \tag{6}$$

where $k = \frac{\omega}{c}$ is the wave number.

And for a certain point x, part of wave will transmit forward while part of wave will be reflected back and the ratio between them is called reflection coefficient R, given by

$$R = p^-/p^+ \tag{7}$$

However, at the end of an open tube, the wave is no longer plane. So the reflection coefficient R_r at the open end is defined based on the coefficient R_Δ at a distance Δ inside from the open end.

$$R_r = R_L e^{2jk\Delta} \tag{8}$$

2.5. End correction

The length of the pipe determines the fundamental frequency of the final sound. However, due to the inertia of the air, the effective acoustical length of the pipe is slightly larger than the length of the pipe. The length correction at the end is a complex and frequency-dependent quantity. Most of the consideration will only consider the real part of the length given by

$$l = Re(k^{-1}arctan[Z_r/(jZ_c)]) \tag{9}$$

where Z_r is the radiation impedance and Z_c is the characteristic impedance. And effective acoustical length is $L + l$, where L is the length of the pipe.

2.6. Simulation parameters

For the simulation, parameters are set at the temperature $t_0 = 25.5937°C$, other parameters are derived correspondingly [7].
Sound velocity: $c = 346.63m/s$,
Density: $\rho = 1.1821kg/m^3$,
Viscosity: $\mu = 1.8348 \times 10^{-5}kg/m \cdot s$,
Thermal conductivity: $\kappa = 0.0063Cal/ms°C$,
Prandtl number: $Pr = 0.7037$,
Ratio of specific heats: $\gamma = 1.4020$.

3. VALIDATION OF FEM

3.1. Finite element method

For studying the input impedance or the reflection coefficient, an interest in frequency properties, of a pipe, Helmholtz equation, a time-independent form of wave equation, is usually used as the governing equation.

$$\nabla^2 p + k^2 p = 0 \tag{10}$$

where $k = \omega/c$ is the wavenumber and c is the speed of the sound.

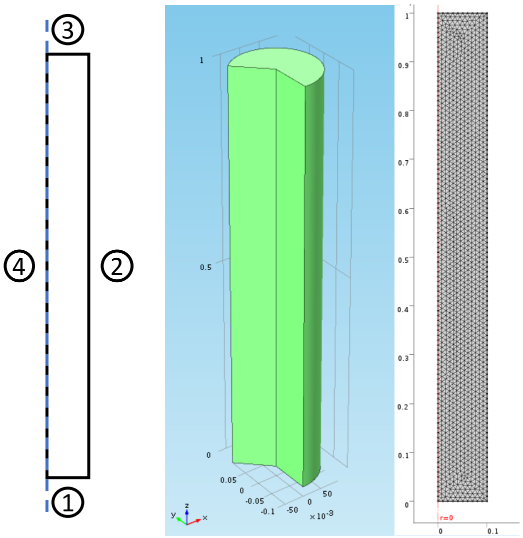
Here, the finite element method is used to solve Helmholtz equation as implemented inside COMSOL.

3.2. Closed cylinder

In order to test the reliability of the simulation result based on COMSOL, the input impedance of a closed cylinder is calculated [8]. In this experiment, a cylinder with radius $a = 0.1\,m$, tube length $L = 1\,m$ is built. In COMSOL, suppose the plane wave propagating inside, a 3D cylinder could be represented and simulated in a 2D symmetrical model. Only half of the cross-section is calculated which saves plenty of time.

As shown in Figure 1a, four boundary conditions need to be given. The air is excited at boundary 1. Boundary 4 is the axial symmetry boundary. Boundary 2 and 3 are set as hard wall boundary are set to boundary first and then a wall impedance $1/Y$ is given to them based on Eq. 2.3;

The meshing is automatically done in COMSOL and 2606 elements is generated in total as shown in Figure 1c.



(a) (b) (c)

Figure 1: (a) Schematic view, (b) 3D view and (c) the meshing of the 2D symmetrical cylinder

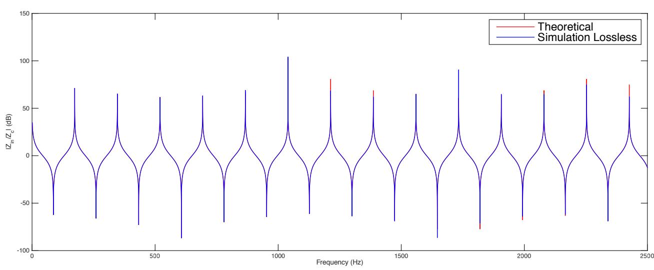


Figure 2: The comparison between theoretical and simulation results for a closed pipe

The result compared with theoretical input impedance is shown in Figure 2. The result is quite similar and the relative error is less than 3%.

In Figure 3, the losses and lossless results is compared. Because of the thermoviscous loss at the wall, there is a obvious loss at each peak of the input impedance.

3.3. Open Cylinder

The input impedance of an open pipe is calculated for the same geometry. Sound soft boundary condition is added to the boundary 3 where the pressure at boundary is set to 0 to simulate an open end effect.

The simulated result is shown in Figure 4 which matches the theoretical one very well and the discrepancy is less than 1%.

4. COMPARISON BETWEEN TWO DIFFERENT MODELS FOR AN OPEN CYLINDER

In this section, the input impedance, radiation impedance and reflection coefficient of two different pipes based on two different models will be studied. The geometries of 2D axisym-

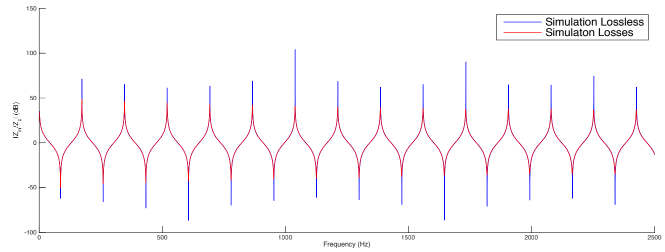


Figure 3: The comparison between losses and lossless simulation results for a closed pipe

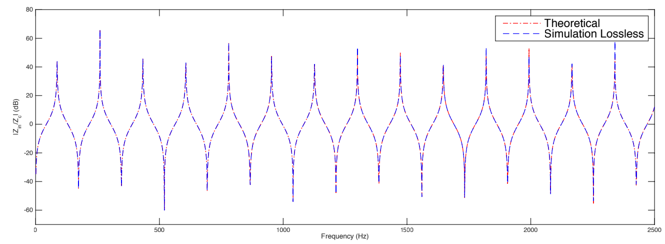


Figure 4: The comparison between theoretical and simulation results for an open pipe

metrical and fully 2D model are shown in Figure 5. Unflanged and flanged geometries are built. The boundary of the radiation area is set as plane wave radiation boundary in COMSOL simulating pressure wave radiating into infinite space. Sound hard wall boundary is set to the boundary of the pipe and the flanged edges.

The radius a and the length L of the pipe is 4 cm and 0.5 m respectively. As discussed above, the wave front of an open pipe is no longer plane, so the acoustic impedance Z_Δ is measured at the observation point located $\Delta = 8a$ from the open end inside the tube as shown in Figure 5. And the reflection coefficient R is deduced from Eq. 8 and R_Δ given by

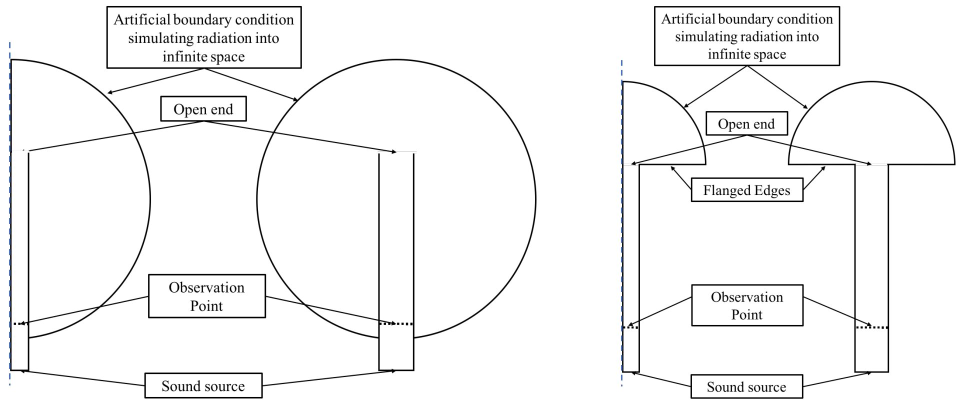
$$R_\Delta = \frac{Z_\Delta - Z_c}{Z_\Delta + Z_c} \tag{11}$$

The radiation impedance Z_r could also be calculated based on Eq. 1. However, based on Dalmont [5], the radiation impedance can also be deduced from Z_Δ given by

$$Z_r = jZ_c tan\left[arctan(Z_\Delta/jZ_c) - k\Delta\right] \tag{12}$$

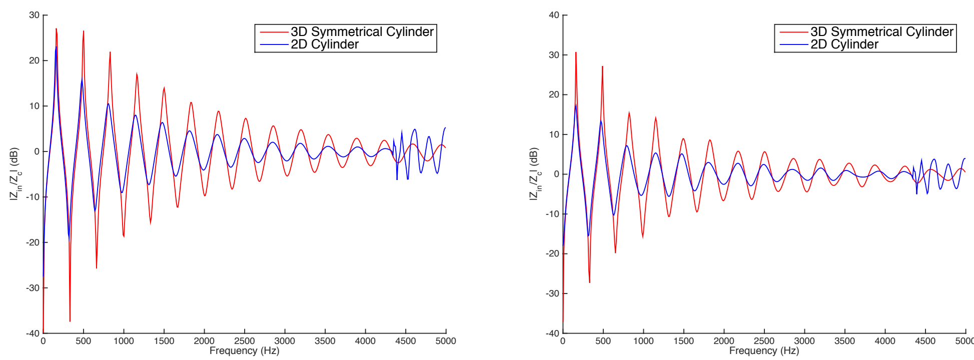
From the Figure 6, the comparison results of input impedance between two different models, the fully 2D cylinder causes a strong damping effect and a small shift along the frequency axis.

From the Figure 7, the comparison of reflection coefficient of two different model, the reflection coefficient of axisymmetrical model overall match theoretical result though there is larger disturbance for the flanged pipes. However, the reflection coefficient of fully 2D cylinder is overall less then the theoretical one which may cause a larger transmission forward outside the pipes. The approximation formula for modulus of the reflection coefficient for unflanged pipe [3] and flanged



(a) (b)

Figure 5: Schematic view of the FEM models for radiation of an symmetrical pipe (a) Unflanged pipe and (b) a flanged pipe. In both figures, the left ones are the 2D axisymmetrical and right ones are two dimensional.



(a) (b)

Figure 6: Comparison between input impedances of (a) an unflanged cylinder and (b) a flanged cylinder

pipes [4] are given by the following two equations respectively

$$|R_o| = \frac{1 + 0.2ka - 0.084(ka)^2}{1 + 0.2ka + (0.5 - 0.084)(ka)^2} \quad for\, ka < 3.5 \tag{13}$$

$$|R_\infty| = \frac{1 + 0.323ka - 0.077(ka)^2}{1 + 0.323ka + (1 - 0.077)(ka)^2} \quad for\, ka < 3.5 \tag{14}$$

The radiation impedance is also shown in Figure 8.

The approximate formula for equivalent length proposed by Norris and Sheng [4] is used as a reference. The form is given by the following equations for unflanged and infinite flanged cylinders respectively:

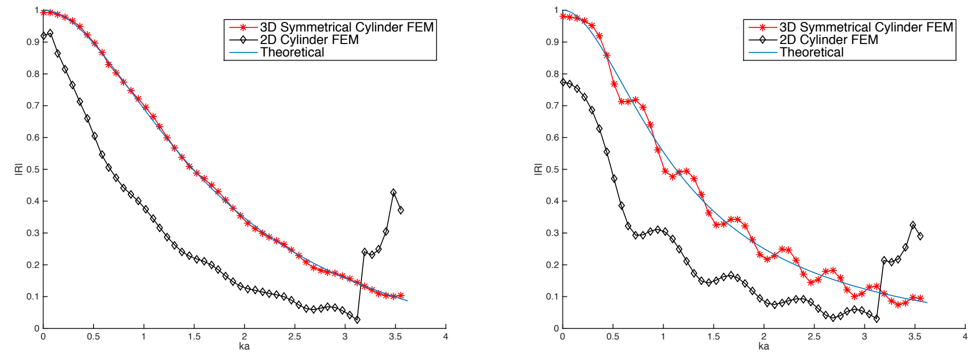
$$l_o = 0.6133a \frac{1 + 0.044(ka)^2}{1 + 0.19(ka)^2} \quad for\, ka < 3.5 \tag{15}$$

$$l_\infty = 0.8216a \left[1 + \frac{(0.77ka)^2}{1 + 0.77ka}\right] \quad for\, ka < 3.5 \tag{16}$$

The comparison for end correction is shown in Figure 9.

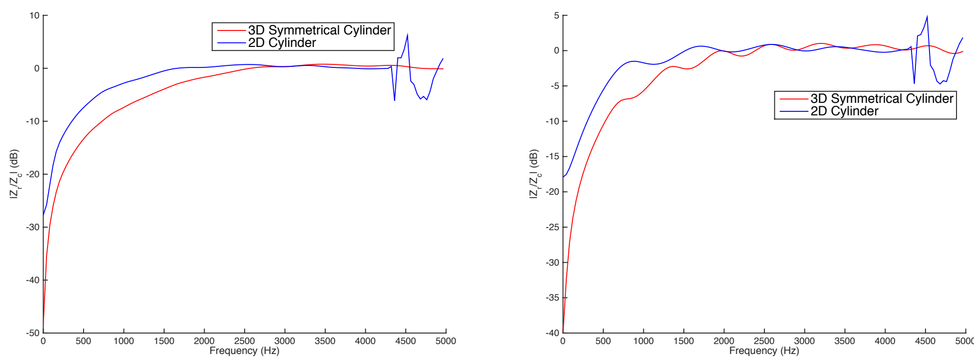
5. CONCLUSION AND FUTURE WORKS

In this paper, the FEM implemented by COMSOL is used to study the differences between 2D symmetrical model and fully 2D model. The method was firstly validated by calculating the input impedance of simple open and closed pipes and the results match the theoretical solution very well. Then it was applied to the unflanged and flanged open pipe where both analytical solution and approximation formula have been studied. For the result, the simulation results based on 2D symmetrical model are more accurate though there is large disturbance on the reflection coefficient of flanged pipe which may be generated by the non-appropriate set boundary condition. The input impedance simulated by the fully 2D model has obvious losses and frequency shift downward compared to the 2D symmetrical one. The reflection coefficient of fully 2D model is overall smaller than the theoretical solution and the comparison between end correction shows the fully 2D result has a slightly longer effective acoustical length which agrees with the result shown in the input impedance. Based on this, we can conclude that using 2D model for the simulation is not accurate and could significantly influence the radiation at the



(a) (b)

Figure 7: Comparison between reflection coefficients of (a) an unflanged cylinder and (b) a flanged cylinder



(a) (b)

Figure 8: Comparison between radiation impedance of (a) an unflanged cylinder and (b) a flanged cylinder

open end and may result in a lower frequency sound.

However, because of the limitation of time in this project and considering it the first time the author using COMSOL and FEM simulation. It is still not 100 percent sure about the result and need further checking and validation. What is more, the disturbance in Figure 7b need to be fixed and more different geometries like different pipe-flange radii ration a/b could be studied. And it will be great if a model could be raised so that 2D simulation could still be used in future research.

6. ACKNOWLEDGMENT

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REFERENCES

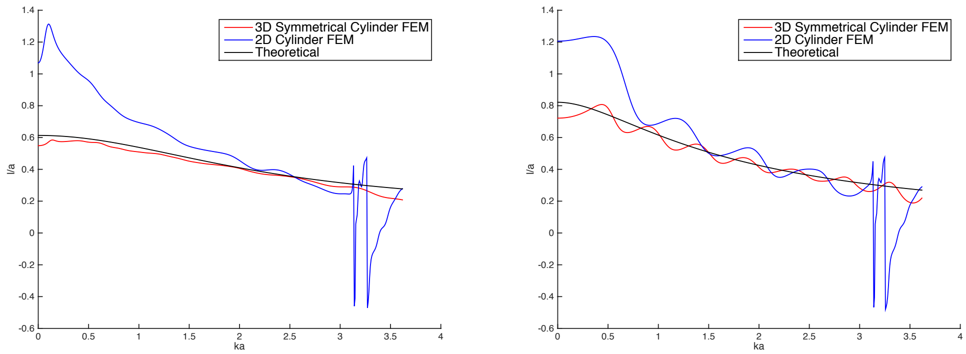
[1] H. Levine and J. Schwinger, “On the radiation of sound from an unflanged circular pipe,” *Physical review*, vol. 73, no. 4, p. 383, 1948.

[2] Y. Nomura, I. Yamamura, and S. Inawashiro, “On the acoustic radiation from a flanged circular pipe,” *Journal of the Physical Society of Japan*, vol. 15, no. 3, pp. 510–517, 1960.

[3] R. Caussé, J. Kergomard, and X. Lurton, “Input impedance of brass musical instrumentscomparison between experiment and numerical models,” *The Journal of the Acoustical Society of America*, vol. 75, no. 1, pp. 241–254, 1984.

[4] A. Norris and I. Sheng, “Acoustic radiation from a circular pipe with an infinite flange,” *Journal of Sound and Vibration*, vol. 135, no. 1, pp. 85–93, 1989.

[5] J.-P. Dalmont, C. Nederveen, and N. Joly, “Radiation impedance of tubes with different flanges: numerical and experimental investigations,” *Journal of sound and vibration*, vol. 244, no. 3, pp. 505–534, 2001.



(a) (b)

Figure 9: Comparison between l/a of (a) an unflanged cylinder and (b) a flanged cylinder

[6] A. R. D. Silva, P. H. Mareze, and A. Lenzi, "Approximate expressions for the reflection coefficient of ducts terminated by circular flanges," *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, vol. 34, no. 2, pp. 219–224, 2012.

[7] A. Chaigne and J. Kergomard, *Acoustics of Musical Instruments*, 2016.

[8] M. Harazi, "A numerical and experimental analysis of the acoustical properties of saxophone mouthpieces using the finite element method and impedance measurements," 2012.

A. PROBLEMS MET IN THIS PROJECT

- As it is the first time using COMSOL and FEM to study acoustic problems, learning the software cost lots of time. The demo in COMSOL called "open pipe" even have errors and the result is inconsistent with the result shown in the documentation.
- The validation model used in Lefebvre's thesis is also built but still could not get the right results yet.
- Two different equations are used to get the radiation impedance Z_r, first is to calculate the reflection coefficient at the observation point and get the R at the open end based on Eq. 8, second is to calculated directly use Eq. 12 given in Dalmont's paper. The first equation could lead to the right result but the second one failed. The equation in Silva's paper [6] is also tried which agreed with the one get by Eq. 8.
- The accuracy of the simulation result is not very satisfied for the unflanged and flanged cylinders, there might be some inappropriate boundary conditions set which need further study.

B. NOMENCLATURE

A= cross section or the cylinder, m^2
a= cylinder radius, m
C_+, C_-= complex constants that describe the amplitude and phase of each traveling-wave component C= specific heat capacity, $Cal/(kg°C$
c= speed of sound, m/s
f= frequency, Hz
j= complex unity $\sqrt{-1}$
k= wave number, m^{-1}
ka= Helmholtz number, dimensionless
L= length of the pipe, m
l= end correction, m
Pr= Prandtl number, dimensionless
p= pressure, Pa
R= reflection coefficient, dimensionless
t= temperature in Celsius, $°C$
U= volume velocity, m^3/s
Y= Admittance
Z= impedance, $Pa \cdot s/m^3$
ρ= density, kg/m^3
κ= thermal conductivity, $Cal/ms°C$
μ= viscosity, $kg/(m \cdot s)$
γ= ratio of specific heats, dimensionless
θ= angle, $°$
Δ= length inside to the pipe from the open end, m