A COMPARISON OF IMPEDANCE MEASUREMENTS USING ONE AND TWO MICROPHONES

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Abstract

Measurements of acoustic input impedance of wind instruments using two different approaches are presented. In the first approach, commonly referred to as the two-microphone transfer function method, a tube is connected to the instrument and excited with broad-band noise. Signals recorded at microphone pairs placed along the tube are then analyzed to estimate the instrument input impedance. A calibration step is described, wherein the position of each microphone pair is determined from the measurement of a rigid termination. The second technique, a novel variant of pulse reflectometry, makes use of a long tube with a single microphone located at its midpoint. Using a long-duration broad-band stimulus, the impulse response is measured for the tube, first with a rigid termination, and then with the system to be characterized attached. The system reflectance, and therefore its impedance, is found by comparing the first reflection from the tube end for both measurements. The design of the impedance probes and the data sampling and analysis procedures are presented. Measurements obtained using the two techniques are compared for various acoustic systems, including an alto saxophone neck and fabricated conical objects. The results show good agreement between the methods. Advantages of the one-microphone technique include ease of use and robustness to noise, while the two-microphone approach can provide a better high-frequency response for long objects.

INTRODUCTION

The measurement of acoustic impedance has been the subject of much research since the beginning of the last century and a great number of publications have been written on the subject. Benade and Ibis (1987) and Dalmont (2001) provide a good background on the historical origins and development of these techniques. Since the 1980s, two measurement techniques have become widely used: the two-microphone transfer function (TMTF) technique and pulse reflectometry.

The use of two microphones located along an acoustic transmission line to evaluate the impedance of an object dates back to the early 19th century (see Beranek, 1988). The two-microphone transfer function technique introduced by Seybert and Ross (1977) made use of a broad-band source signal and Fourier analysis to evaluate the impedance over the entire spectrum in one measurement. It has also been described by Chung and Blaser (1980a,b).

Pulse reflectometry originated from geophysical studies of the earth's crust but, throughout the 1970s and 1980s, it was applied to the study of the vocal tract (see Fredberg *et al.*, 1980) and to musical instruments. The novel approach reported here is based on the same principle as pulse reflectometry but achieves an improved signal-to-noise ratio (SNR) by using wide-band signals of long duration, such as swept sines. For the purposes of this paper, we shall refer to this technique as "impulse reflectometry" (IR).

The objective of this paper is to compare impedance measurements obtained with both techniques in order to identify and characterize possible discrepancies between the two, as well as to better assess the accuracy of the results and the importance of measurement errors. In the context of musical acoustics, we are mainly interested in the magnitudes and frequencies of the maxima and minima of strongly resonant bodies.

We first detail the experimental setup, calibration procedures, and signal analysis methods for both techniques. We then present impedance measurement results for three objects: an alto saxophone neck, a short carbon fiber cone, and a long carbon fiber cone coupled with the neck. We conclude with a comparison of the advantages and disadvantages of both techniques.

THE TWO-MICROPHONE TRANSFER FUNCTION TECHNIQUE

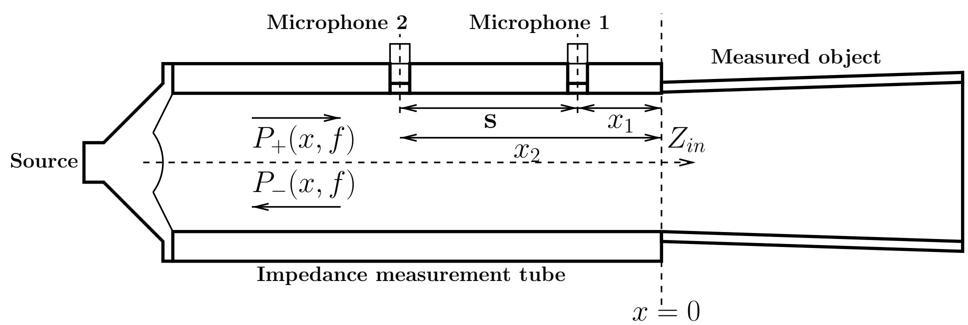

Figure 1. Diagram of the two-microphone measurement apparatus.

In the two-microphone transfer function technique, the impedance of an object is evaluated from the measurement of the transfer function between two microphones located at different positions along a waveguide connected to that object. A horn driver emits a broad-band signal, such as white noise, in the waveguide over a time duration adequate to reduce variance in the results, as computed with a modified average periodogram.

This technique is based on the mathematical theory of one-dimensional planar pressure wave propagation in a cylindrical duct. Such waves, including attenuation, can be described by the equation

$$P(x,f) = P_+(x,f) + P_-(x,f) = Ae^{-\Gamma x} + Be^{\Gamma x}, \tag{1}$$

where A and B are the complex frequency-dependent amplitudes of the progressive and regressive traveling-wave components. The propagation parameter is defined as $\Gamma = \alpha + i\omega/v_\phi$, where α is the attenuation and v_ϕ the phase velocity. Estimation of this parameter has been described by Pierce (1989). It can be approximated by $\Gamma = i\omega/c + (1+i)\alpha$, where $\alpha \propto \sqrt{f}$ by a constant that depends on air properties.

From these equations, it can be shown (Lefebvre, 2006) that the impedance $\bar{Z}_{in}$ of an object located at $x = 0$ (see Fig. 1) is given by

$$\bar{Z}_{in} = \frac{Z}{Z_c} = \frac{H_{12}\sinh(\Gamma x_1) - \sinh(\Gamma x_2)}{H_{12}\cosh(\Gamma x_1) - \cosh(\Gamma x_2)}, \tag{2}$$

where H_{12} is the transfer function between the two microphones and Z_c is the characteristic impedance.

This approach is based on one-dimensional wave propagation and thus, it is limited in frequency to the first higher-order mode that occurs at $f = 1.84c/(2\pi r)$, where r is the cylinder radius and c is the speed of sound. For our measurement system, the cutoff frequency is approximately 16.5 kHz ($r = 0.006$ meters). The TMTF technique is also incapable of providing results

Microphone Pair	Distance	Frequency Range (Hz)
1 and 2	3 cm	575 - 4600
1 and 3	12 cm	290 - 1150
1 and 4	36 cm	95 - 380

Table 1. microphone pairs use in our measurement apparatus

at critical frequencies where the two pressure signals become linearly dependent, which equates to half-wavelengths that are an integer multiple of the microphone spacing:

$$f_c = mc/2s, m = 1, 2, ..., N. \tag{3}$$

The consequence is that we need several pairs of microphones to cover a sufficient frequency range for musical instrument characterization. To achieve a frequency range of 100 – 5000 Hz, we use four microphones. Table 1 indicates the microphone distances and valid frequency ranges. Final impedance results are realized by concatenating impedances from three microphone pairs.

Prior to the measurement, a relative calibration of microphones pairs is performed, as described by Seybert and Ross (1977) and Krishnappa (1981), in order to eliminate frequency response differences between them. This calibration is made using a special apparatus such that the four microphones are located at the same reference plane and exposed to a broadband noise signal. The microphone positions used in Eq. (2) can also be fine-tuned with a measurement obtained when the plane at $x = 0$ is rigidly terminated. For this condition, the transfer function between two microphones is given by (see Lefebvre, 2006):

$$H_{12} = \frac{\cosh(\Gamma x_2)}{\cosh(\Gamma x_1)}. \tag{4}$$

The attenuation parameter, α, which will be higher than predicted if the tube inner surface is not completely smooth, can also be calibrated from the magnitudes of these maxima and minima.

We evaluate the transfer function H_{12} between the recorded signals at the two microphones with the total least square formulation, which reduces the impact of noise (see P.R. White, 2006):

$$H_{12} = C_{12} \cdot \frac{S_{p_2p_2} - S_{p_1p_1} + \sqrt{(S_{p_1p_1} - S_{p_2p_2})^2 + 4|S_{p_1p_2}|^2}}{2S_{p_2p_1}} \tag{5}$$

where $S_{p_1p_1}$ is the auto-correlated spectral density of the first microphone signal, $S_{p_1p_2}$ is the cross-correlated spectral density between microphones 1 and 2, etc. C_{12} is the calibration function previously measured.

IMPULSE REFLECTOMETRY

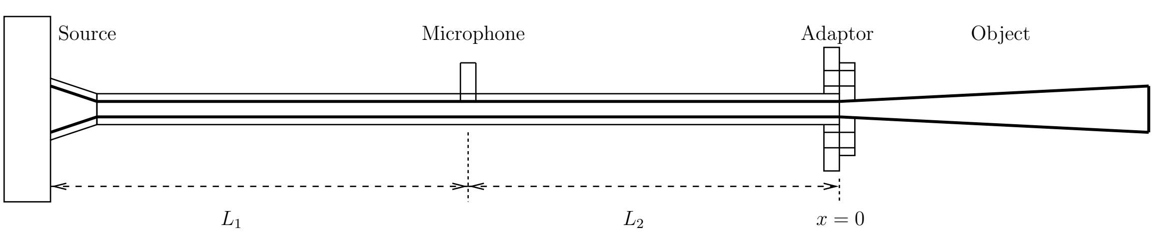

Figure 2. Setup for the one-microphone measurement system.

The impulse reflectometry (IR) technique uses a setup with a single microphone and a calculation based on two measurements. The apparatus consists of a horn driver connected to a long probe tube and a microphone located near its midpoint, as illustrated in Fig. 2. After performing a measurement with the probe tube rigidly terminated, the object to be measured is attached to the end of the probe and another measurement is made.

In contrast to traditional pulse reflectometry techniques, a long duration source signal, such as a swept sine, is used to make the measurements. The benefit is that a lot of energy can be supplied to the system, increasing the measurement signal-to-noise ratio. In the measurements reported here, a logarithmically swept sine was used. The impulse response of the system fitted alternately with a rigid termination and with the object of interest is measured by deconvolving the recorded signal, $y(t)$, from the input or source signal $x(t)$:

$$ir(t) = \Re\left\{\text{IFFT}\left[\frac{\text{FFT}(y(t))}{\text{FFT}(x(t))}\right]\right\}. \tag{6}$$

This approach assumes our objects of interest are linear and time-invariant. Because the source signal $x(t)$ is non-zero at all frequencies of interest, there are no stability problems with this calculation. Each impulse response then consists of a series of pulses corresponding to an initial pulse (p_1) from the driver, its reflection from the reference plane at $x = 0$ (p_2), the reflection back from the driver (p_3), etc. The reflection coefficient $R_{in}(f)$ of the measured object is evaluated by taking the ratio of the Fourier transform of the time-windowed first reflection from the object, FFT(p_{2o}), and the Fourier transform of the time-windowed first reflection from the rigid termination, FFT(p_{2r}):

$$R_{in}(f) = \frac{\text{FFT}(p_{2o})}{\text{FFT}(p_{2r})}. \tag{7}$$

The normalized input impedance of the object is then calculated as $\bar{Z}_{in} = (1 + R_{in}) \,/\, (1 - R_{in})$.

As with traditional pulse reflectometry, it is necessary that the impulse response of the object to be measured be shorter than the corresponding propagation time along one length of the probe tube. Alternately, the impulse response must decay in time before its reflection from the driver returns to the microphone position. The consequence is that a longer measurement tube is needed to measure objects with long impulse responses. As has already been pointed out by Sharp (1996, pg. 84), the use of a longer probe implies more propagation losses and, because losses increase with frequency, a reduced frequency range. This study made use of two different probe tubes: a straight aluminum pipe of 5 m length and 0.015 m diameter, referred to as IR (straight); and a coiled copper pipe of 18 m length and 0.0127 m diameter, referred to as IR (coil).

RESULTS

Three objects were measured for this study as follows:

1. a Selmer series II alto saxophone neck ($r_1 = 6.30$ mm, $r_2 = 11.35$ mm, $L = 195$ mm);
2. a short carbon fiber cone ($r_1 = 6.15$ mm, $r_2 = 16.60$ mm, $L = 402$ mm);
3. a long carbon fiber cone ($r_1 = 11.75$ mm, $r_2 = 36.0$ mm, $L = 834$ mm) coupled with the neck.

All the measurements made use of a JBL 2426H compression horn driver, Sennheiser KE4-211-2 omni-directional electret microphone capsules, and an RME Fireface 800 audio interface for signal output and acquisition. The microphone capsules were amplified with a circuit based on the AD822A operational amplifier for the TMTF probe, while a Unides Design conditioner was used for the IR measurements.

The measured input impedance magnitude results are plotted in Figs. 3–5. The top plot in each figure shows the unprocessed results, while the lower plots are compensated for temperature differences and probe tube discontinuities as described below. Each object was measured using the TMTF technique, as well as IR with the two different probe tube lengths mentioned above. For comparison, theoretical values are also included in the plots as calculated using frequency-domain transmission-line theory (Caussé *et al.*, 1984). Each object was assumed to be well represented by a single conical waveguide section. To achieve a good approximation for the boundary layer losses, each conical segment was divided into small (roughly 1 cm) concatenated sections so that the median radius values used in the calculations were better approximated.

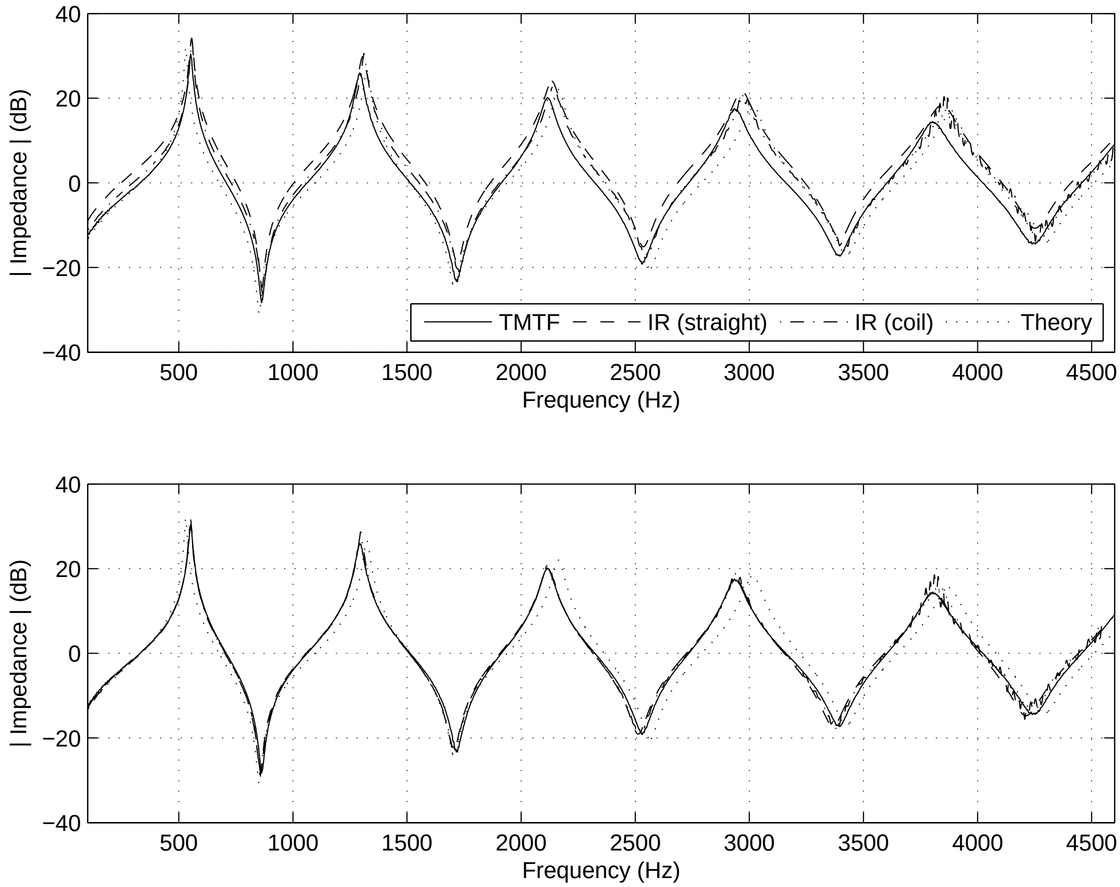

Figure 3. Input impedance magnitude of the alto saxophone neck: raw (top) and compensated (bottom).

While all the measurements results are relatively close to one another, discrepancies are evident. We recognize several potential sources for these discrepancies, including variations in temperature, probe tube diameter, and inherent limitations of the measurement techniques.

Ideally, the various measurements should be made under equal and constant atmospheric conditions. Variations in temperature and barometric pressure can cause variations in sound speed and also affect propagation loss characteristics. The air temperature was $24.6°$C for the TMTF measurements, $22.4°$C for the IR (coil), and $22.6°$C for the IR (straight). Our results were compensated (lower plots in each figure) by scaling the frequency axis by a factor proportional to the ratio of the speed of sound for the measurement to a reference speed. The frequency scaling factor is about 0.997 to normalize the IR and TMTF measurements.

The single most important source for discrepancies in the measurements can be related to variations in the impedance probe diameters. Ideally, the probe should have the same diameter as the input of the object to be measured so as to minimize the excitation of evanescent modes at a discontinuity. While the TMTF probe met this condition, diameter discontinuities existed for both IR probes (and was most significant for the 5 m straight pipe). A method to correct for reference-plane discontinuities is discussed by van Walstijn *et al.* (2005) for cases where the measured object input is cylindrical. All of the objects measured for this study have conical inputs, thus preventing application of that technique. Interestingly, we found that empirically determined magnitude offsets of -4.0 dB and -1.85 dB for the straight and coiled IR data, respectively, produced a good fit

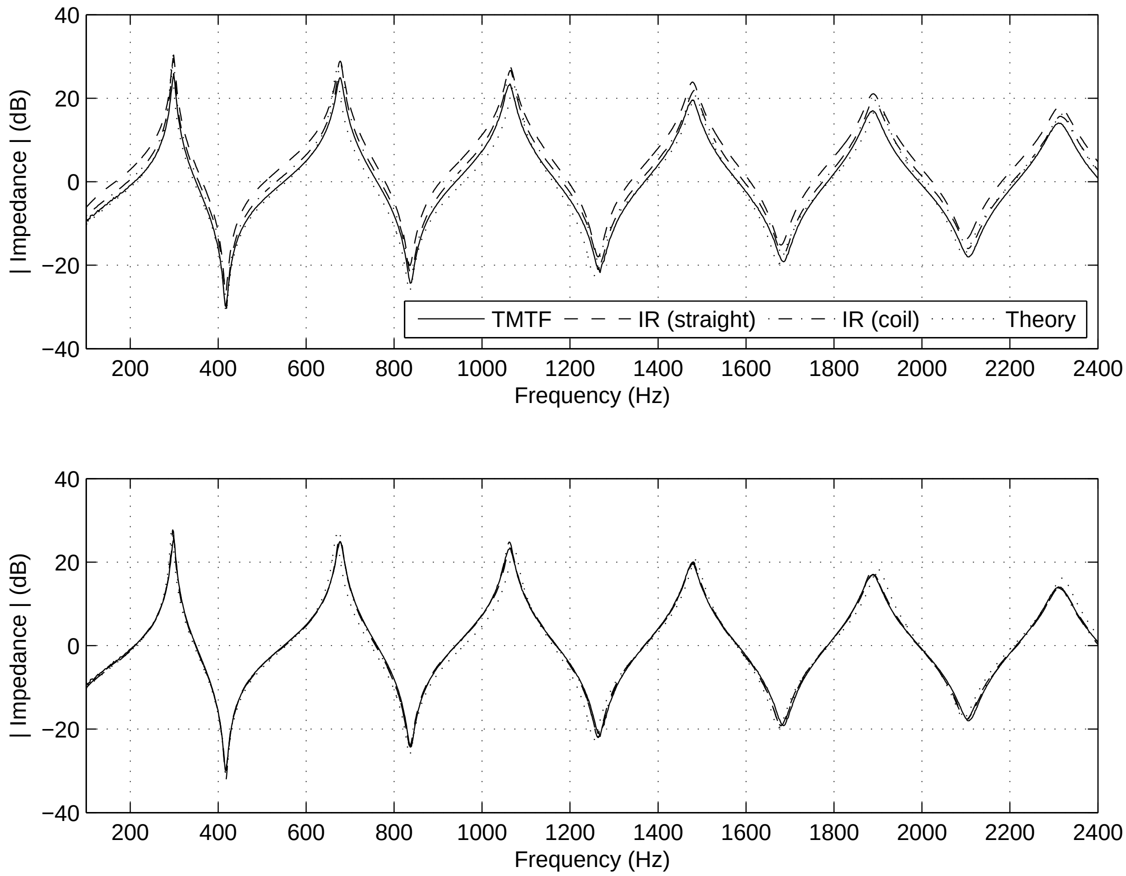

Figure 4. Input impedance magnitude of the short carbon fiber cone: raw (top) and compensated (bottom).

to the TMTF results (which did not have a diameter discontinuity). The compensated lower plots in each figure include these offsets.

As previously mentioned, the impulse reflectometry technique is susceptible to high-frequency attenuation when using long probes, thus reducing the SNR. In the impedance measurement, the lower SNR manifests itself as noise. An instance of this behavior is noticeable above $\approx$ 2.5 kHz in Fig. 3 for the IR (coil) data. By using a source signal with a different frequency trajectory, for instance a linear sweep, more high-frequency energy could be supplied to the system and a better SNR achieved. We see from Fig. 5 that the measurement of the long cone is not possible with the IR (straight) probe. In that case, the impulse response of the object is too long to be adequately resolved by the 5 meter pipe.

Though difficult to distinguish in the figures, we observe that the two-microphone transfer function technique provides noisier results and that this noise is stronger at the maxima and minima of the impedance. Those extrema are smoother with the IR technique and, especially with the coil, they are slightly greater (about 2dB) and closer to the theoretical predictions. We can also observe that the match between the impedance results made with the three microphone pairs is quite good, which means that the calibration procedure works well.

Although the TMTF technique produces good results and the match between the concatenated impedances is nearly perfect, it remains tricky to use correctly. The major problem is that the probe has many resonances due to its short length and the reflectivity of the horn driver. This causes relatively large variations in the amplitudes of the signals at the resonances and anti-resonances of the system. When the signals are strong, there is a risk of distortion in the microphones. Conversely, when the signal amplitudes are low, the SNR ratio can be quite poor, which reduces the quality of the results. The same problem appears with the microphone calibration apparatus. Thus, the TMTF driver and microphone gains are difficult to set properly. It is also necessary to make

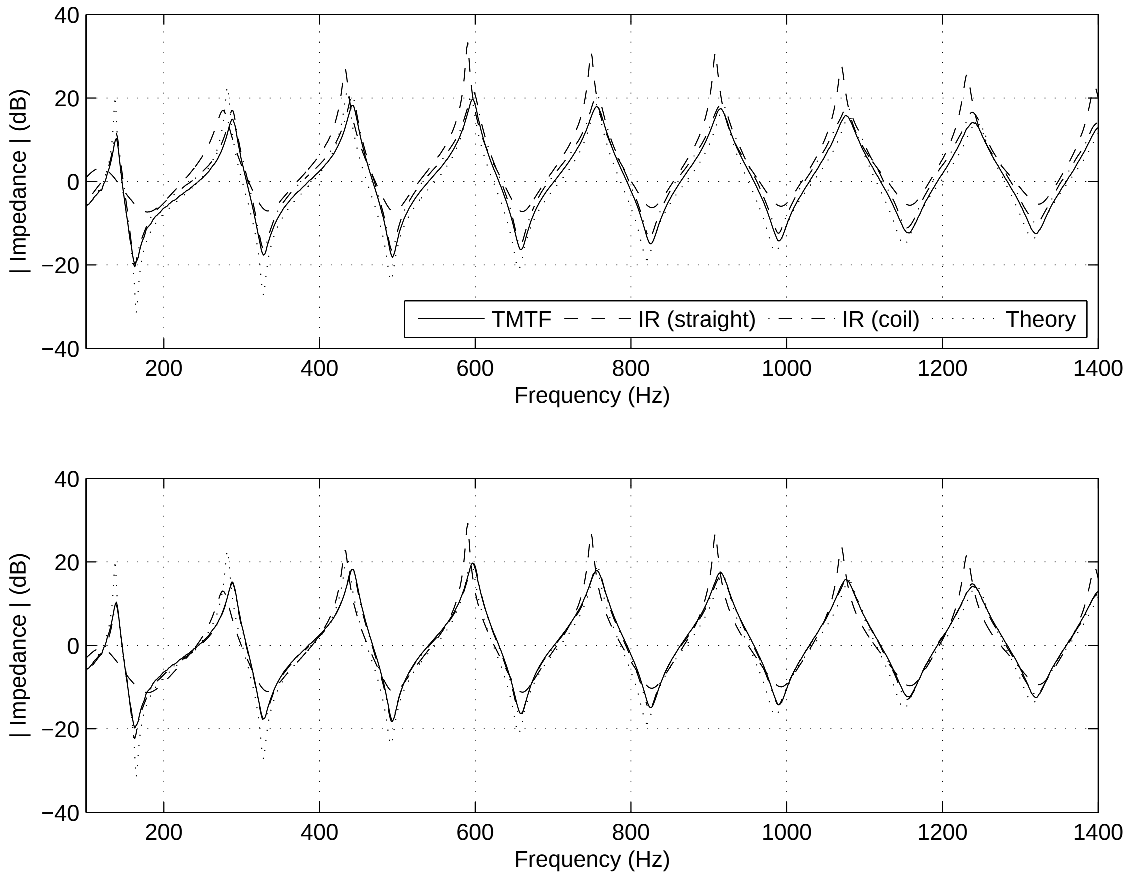

Figure 5. Input impedance magnitude of the long carbon fiber cone and saxophone neck: raw (top) and compensated (bottom).

the TMTF measurements in a low noise environment, whereas the IR technique works well in a fairly noisy computer lab environment.

When compared to theory, the compensated measurements for the short carbon fiber cone are closest. The theoretical values are slightly lower in frequency for the first two maxima (-17 and -21 cents, respectively) but a bit higher in frequency for the third maxima (+16 cents). The predicted frequencies are slightly lower for all impedance minima except the first. The theory appears to underestimate the losses in the system. For the saxophone neck, the theoretical value is lower in frequency for the first maxima (-64 cents) and higher for the other maxima (+36, +40, . . ., cents). The trend is similar for the impedance minima. Discrepancies for the saxophone neck are to be expected because the theoretical values were calculated for a perfect conic section whereas the neck is curved and has a closed register hole. For the long carbon fiber cone and saxophone neck combination, there is a significant magnitude difference (about 10 dB) between the measurements and the theoretical values of the first few impedance maxima. Again, this appears to indicate that losses are underestimated by the theory. The predicted and measured maxima and minima frequencies are within about 20–30 cents at low frequencies and differ by less than 10 cents for the third and higher minima.

CONCLUSIONS

In comparing these two measurement techniques, there are clear advantages and disadvantages to each. The impulse reflectometry approach requires only one microphone and no calibration. The measurement results have very smooth impedance maxima and minima and appear to be closer to the theoretical values. A significant disadvantage of the IR technique involves the need for a long probe tube, making it less portable. Further, there is a compromise between the size of the object to be evaluated and the highest possible frequency that can be measured. The TMTF technique uses a more compact apparatus and easily allows for high-frequency impedance measurements (up to the first higher-order mode). However, this approach requires multiple microphones and precise calibration steps that significantly increase the necessary setup time. Further, TMTF results at impedance minima and maxima tend to be somewhat noisy.

Future work will involve new measurements with an IR probe tube of the same inner diameter as the input of the objects to be measured, as well as experiments with different driver and microphone positions. A technique is also being explored that halves the required IR probe length. Finally, we plan to investigate the use of "designed" chirps with the TMTF technique to compensate for resonances in the system and reduce noise in the results.

REFERENCES

Benade, A. H. and Ibis, M. I. (**1987**). "Survey of impedance methods and a new piezo-disk-driven impedance head for air columns", J. Acoust. Soc. Am. **81**, 1152–1167.

Beranek, L. L. (**1988**). *Acoustical Measurements* (Acoustical Society of America).

Caussé, R., Kergomard, J., and Lurton, X. (**1984**). "Input impedance of brass musical instruments-comparison between experiment and numerical models", J. Acoust. Soc. Am. **75**, 241–254.

Chung, J. Y. and Blaser, D. A. (**1980**a). "Transfer function method of measuring in-duct acoustic properties. I. Theory", J. Acoust. Soc. Am. **68**, 907–913.

Chung, J. Y. and Blaser, D. A. (**1980**b). "Transfer function method of measuring in-duct acoustic properties. II. Experiment", J. Acoust. Soc. Am. **68**, 914–921.

Dalmont, J.-P. (**2001**). "Acoustic impedance measurement, Part I: Review", J. of Sound And Vib. **243**, 427–439.

Fredberg, J. J., Wohl, M. E. B., Glass, G. M., and Dorkin, H. L. (**1980**). "Airway area by acoustic reflections measured at the mouth", J. Appl. Physiol.: Respirat. Environ. Exercise Physiol. **48**, 749–758.

Krishnappa, G. (**1981**). "Cross-spectral method of measuring acoustic intensity by correcting phase and gain mismatch errors by microphone calibration", J. Acoust. Soc. Am. **69**, 307–310.

Lefebvre, A. (**2006**). "La mesure de l'impédance acoustique du saxophone alto", Master's thesis, École Polytechnique de Montréal.

Pierce, A. D. (**1989**). *Acoustics, An Introduction to Its Physical Principles and Applications* (Acoustical Society of America).

P.R. White, M.H. Tan, J. H. (**2006**). "Analysis of the maximum likelihood, total least squares and principal component approaches for frequency response function estimation", Journal of Sound and Vibration **290**, 676–689.

Seybert, A. F. and Ross, D. F. (**1977**). "Experimental determination of acoustic properties using a two-microphone random-excitation technique", J. Acoust. Soc. Am. **61**, 1362–1370.

Sharp, D. B. (**1996**). "Acoustic pulse reflectometry for the measurement of musical wind instruments", Ph.D. thesis, University of Edinburgh.

van Walstijn, M., Campbell, M., Kemp, J., and Sharp, D. (**2005**). "Wideband measurement of the acoustic impedance of tubular objects", Acta Acustica united with Acustica **91**, 590–604.