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RHYTHMIC SIMILARITY: A THEORETICAL AND EMPIRICAL APPROACH

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ABSTRACT

This paper introduces a novel form of representing musical durations (which we will call **chronota**) of melodies in from of chains based on atomic beats. This is, a melody consisting of quarter, eighth and sixteenth notes fetches the sixteenth as its atomic beat, where all other durations are represented as multiples of the sixteenth notes. This form of representation makes it possible to depicted musical durations geometrically in a 2-dimensional space. Depicting two rhythms (sequences of durations called **chains** here) in this representational system, rhythmic similarity (called **chronotonic** similarity here) is seen as being correlated to how much two rhythms deviate in shape via a transformation mechanism. A similarity model based on this form of representation predicts specifically the following scenarios. (A) a quarter note compared to two eighths notes (split ratio 1:1) fetches smaller similarity ratings than a quarter note compared to a dotted eighth note and a sixteenth note (split ratio 1:3), (b) reversing two sequences produces the same similarity rating as the original sequences (c) longer sequences fetch higher similarity ratings, (d) tempo change affects similarity ratings, (e) comparison order has no effect, and (f) complex sequences compared with simple sequences fetch low similarity ratings. The model was put to its test within an experiment. It was found that the model appears to be supported by the results as obtained in this experiment. In fact inputting all data of the experiment into the model, produced a high correlation. Moreover, all predictions of the model were supported by the experiment except of one trial (comparison order) which fetched small negative significance.

1. INTRODUCTION

Interest in melodic similarity has seen a sharp increase within the last years. The publication of "Melodic Similarity: Concepts, Procedures, and Applications" (ed. Hewlett & Selfridge-Field, 1999) as well as the dedication of volume 18.3 of *Music Perception* to the topic of similarity, categorization and segmentation (e.g. Deliege's article on similarity perception) in 2001provide evidence just as much as many of the contributions during the last ISMIR in Bloomington in 2001 (e.g. Hoos, Renz, Görg, presenting a similarity model based on transition matrices).

This interest in melodic similarity might at first seem unmotivated, but considering the importance of this issue becomes more obvious if we look the great many amount of contexts where similarity judgements are of importance: (a) A composer endeavouring to produce a variation to a theme, (b) an ethnomusicologist trying to classify melodic material, (c) a music teacher assessing how close a student's performance is to an appropriate interpretation, (d) a user of a musical data bank in the effort to retrieve a specific melody, and (e) a judge trying to settle a copy-right infringement suit.

However, as much as the issue of similarity is of interest, and however much attention has been paid to it, there exists a wide gap between the cognitive science community and the community of scientists who have been attempting to construct similarity models. This is, while cognitive scientists tend to obtain similarity scales via multi-dimensional scaling without being able to extract melodic features which could be inputted into a model (e.g. Lamon & Dibben, 2001; Eerola, Järvinen, Louhivuori & Toivianinen, 2001), music information retrieval researchers tend to construct models without considering cognitive aspects (compare Hofmann-Engl, 2001). Moreover, we find that no existing model integrates rhythmic similarity and very few cognitive studies investigate rhythmic similarity.

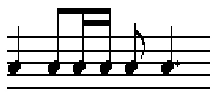
It is the goal of this paper to construct a rhythmic similarity model which will be put to its test within an experiment with the intention to bridge the gap between the cognitive science and the modelling community. This model will be based on a novel representation of musical durations and the transformation mechanism in form of reflections. Thus, the issue of musical representation will be the first step.

2. REPRESENTING MUSIC

The fact that the way we represent music will have an impact on how music will be composed, analysed and understood has been observed (e.g. Selfridge-Field, 1999). In fact, the author claims, the main reason why the construction of a rhythmic similarity model has not yet been attempted, is the inadequate description of musical durations. Let us consider the typical abstract representation of music in form of a sequence of tones (compare Wiggins, Mitch, Smaill, 1989). For reasons of clarity we will call a sequence of tones a ***chain*** and a musical duration will be called a ***chronoton***. Thus, we are not dealing with rhythms in a strict sense, as we are not considering issues such as expressive timing and dynamic accentuations. We write: $ch = [t_1, t_2, ..., t_n]$, where each tone consists of a pitch, a loudness and a chronoton. Thus, a tone at the place i of the chain can be written as: $t_i = \{p_i, l_i, c_i\}$. This representation suggests a sequence of events or elements and implies some concept of time as an event at the place i will be followed by an event at the place i+1. However, each event itself unfolds in time according to the chronotonic values (= musical durations) of the chain and is not equivalent to the time scale as established by the order of the sequence. The author argues that this has to be seen as the fundamental error when representing chronota in form of a sequence. A representation in form of atomic beats as described below endeavours to tackle this issue.

2.1 Notation in Atomic Beats

In order to introduce the concept of atomic beats, it seems most appropriate to consider an example. Let us consider to chrontonic chain (sequence of durations) given as:



This chronotonic chain can be written as: *ch* = [1/4, 1/8, 1/16, 1/16, 1/8, 3/8]. Now, we find that the shortest time value in this chain is the 1/16th note. Quantizing the time into atomic beats of 1/16th beats, we obtain from the first quarter to the last quarter 16 atomic beats. Further, we find that on the first atomic beat there is a quarter note which lasts for the next three atomic beats, on the 5th atomic beat is a 1/8th note which last for another atomic beat and so on. Further, considering that a quarter is four times longer than a 1/16th note, and that a quaver is twice as long as a 1/16 note, we write:

ch = [4, 4, 4, 4; 2, 2; 1; 1; 2, 2; 6, 6, 6, 6, 6, 6](1/16)

This is to be read in the following manner: The atomic beat of the chain is a 1/16th note. The first to the fourth beat fetch the value 4 · 1/16 = 1/4 (quarter note). The fifth and sixth atomic beats fetch the value 2 · 1/16 = 1/8 (eighth note) and so on. We obtain figure 1:

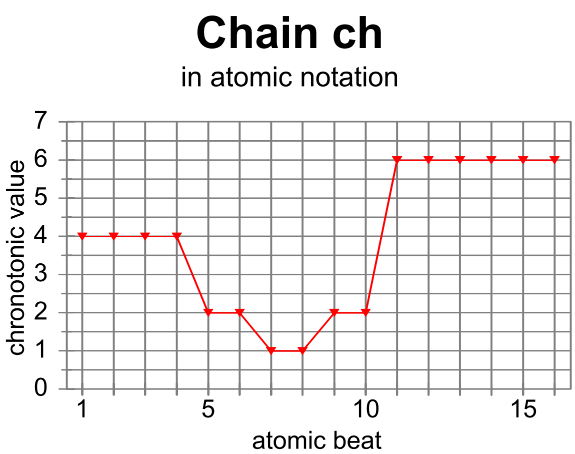
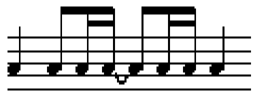


Figure 1: The chain *ch* = [1/4,1/8, 1/16, 1/16, 1/8, 3/8] in atomic notation. The higher the value of a chronoton (the longer the durations), the higher the lines. The length of a line does not necessarily correlate to the chrontonic value, as two consecutive 1/16 notes produce a line as long as 1/8.

As we will see, this form of notation will allow for the transformation of one chain into another chain and ultimately for the development of a similarity measure based on this transformation mechanism.

3. CHRONTONIC TRANSFORMATIONS

We will introduce chrontonic transformations by referring to our chain *ch* as given above and comparing it to a second chain *ch*'. Let *ch*' be:



In atomic notation, we obtain: *ch*' = [4, 4, 4, 4; 2, 2; 1; 3, 3, 3; 1; 1; 4, 4, 4, 4](1/16). It will exceed the framework of this paper to explain the motivation for introducing the following transformation mechanism, but the author hopes, that some of it might become apparent as the text develops.

In order to map *ch* onto *ch*', we reflect *ch* (dark red) along the x-axis, and construct the midpoints (light blue) between *ch*'(dark blue) and the image (light red) of *ch* as illustrated in figure 2, thereby reflecting each point of the image of *ch* onto each point of *ch*'.

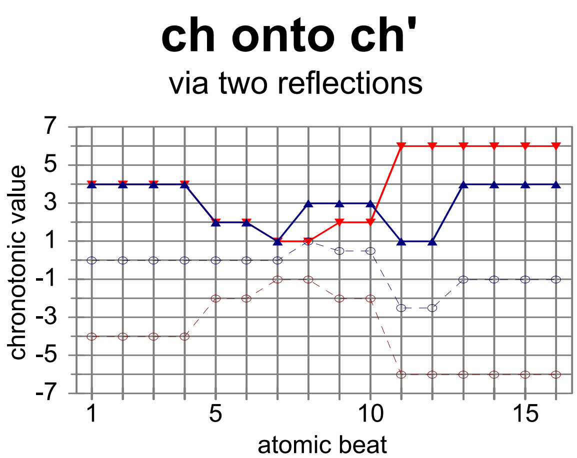


Figure 2: Here, the chain *ch* = [1/4, 1/8, 1/16, 1/16, 1/8, 3/8] (dark red) is reflected along the 0-axis producing its image (light red). Each atomic beat of the image is reflected along a chain of reflection points (light blue) onto the chain *ch*' = [1/4, 1/8, 1/16, 3/16, 1/16, 1/16, 1/4] (dark blue).

Clearly, this transformation mechanism allows us to map any chain onto any other chain as long as they have the same length. But even if two chain have different length, all which is required is to augment the shorter chain so as to match the longer chain. Once the two chains have the same length, the mechanism will be the same as described

This transformation mechanism is best understood as the composition of two reflections within a *n*+1 dimensional space, where *n* is the amount of atomic beats. However, for our purpose

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the above figure will suffice. As we will see in the next paragraph, the reflection chain in figure 2 (light blue) contains the information needed to determine how similar the two chains (rhythms) are.

4. CHRONOTONIC SIMILARITY

One more time, we will consider our example from above, where we intend to compare *ch* = [1/4, 1/8, 1/16, 1/16, 1/8, 3/8] with *ch'* = [1/4, 1/8, 1/16, 3/16, 1/16, 1/16, 1/4] by considering figure 3 which is a modification of figure 2.

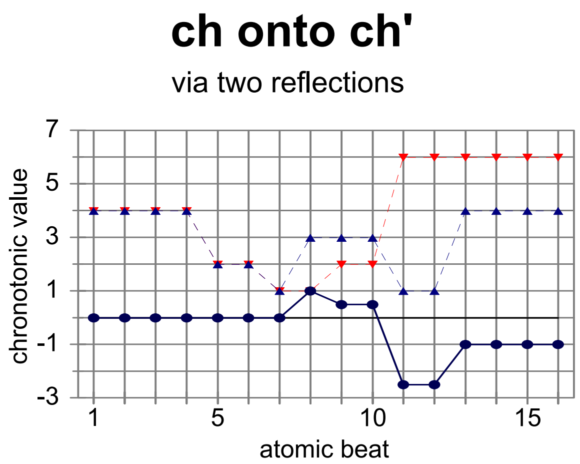

Figure 3: The reflection chain *R* (dark blue circles) represents the degree of similarity of the compared chains. The correlation is: The closer the reflection line to the 0-axis (black line), the higher the degree of similarity.

The degree of similarity between the two chains is given by the closeness of the reflection chain *R* = [0, 0, 0, 0, 0, 0, 0, 1, 0.5, 0.5, -2.5, -2.5, 1, 1, 1, 1](dark blue) and the 0-axis (black line): The smaller the distance of the reflection chain from the 0-axis, the higher is the degree of similarity. In fact, we find for the first 7 atomic beats that the reflection line coincides with the 0-axis, hence both chains must be identical. This is true indeed. Further, we find that both chains are least similar at the atomic beats 11 and 12. This is, where *ch* consists of a dotted quarter and *ch'* of two 1/6th notes. This seems intuitively plausible.

Now, in order to test the hypothesis that the reflection chain is a similarity predictor indeed, the following similarity model was devised:

Firstly, we required for two chains of different length (such as a chain consisting of 4 quarter compared to a chain of 4 eighth notes) to be equalised. Let us assume, that in order to equalise two chains we have to multiply the chronota (musical durations) of one chain by the factor *a* (in case that eighth notes are augmented to quarter notes, *a* will be *a* = 2). As we do not wish to make our model depended on whether we augment or diminute the chains, we write in analogy to Shepard's approach (1987):

$$F_1 = e^{-k_1 \ln^2 a}$$

where F_1 is the similarity predictor where the length of two chains is different, with $|ch| = a\,|ch'|$ and k_1 an empirical constant.

Secondly, in order to capture the similarity predictor correlated to the reflection chain $R = [r_1, r_2, ..., r_n]$ the reader might be reminded that reflections are executed in a n+1 dimensional space, where the reflection chain is depicted as a vector. Here we find that the higher the similarity the longer the vector. Hence, we write:

$$\left\|\vec{F}_2\right\| = \left\| \begin{pmatrix} \frac{e^{-\frac{k_2}{n} r_1^2}}{\sqrt{n}} \\ \frac{e^{-\frac{k_2}{n} r_2^2}}{\sqrt{n}} \\ \cdot \\ \cdot \\ \frac{e^{-\frac{k_2}{n} r_n^2}}{\sqrt{n}} \end{pmatrix} \right\| = \sqrt{\frac{\sum_{i=1}^{n} \left(e^{-\frac{k_2}{n} r_i^2} \right)^2}{n}}$$

where $\left\|\vec{F}_2\right\|$ is the similarity predictor correlated to the reflection chain, n the length of the chains in atomic beats (after equalisation), r_i the ith component of the reflection chain and k_2 an empirical constant.

The chrontonic similarity will be expressed as:

$$S = F_1 \left\|\vec{F}_2\right\|$$

This hypothesis was tested in an experiment.

5. EXPERIMENT

This experiment was designed to test the hypothesis that *S* will appear as a significant predictor in the context of chronotonic (rhythmic) similarity. Tested factors were: Tempo change (a specific chain played twice at different speed/tempo), change of split ratio (a chronoton of a chain split into a ratio 9:1, 7:3 and 1:1), change of length of the chain (by repeating a specific pattern once or twice), reversal of pattern (e.g. dotted 1/8 - 1/16 reversed to 1/16 - 1/8) and comparison of a simple chain with a complex one.

5.1 Method

Equipment

A personal computer Commodore C64, headphone Gamma LH 926 and a Grundig amplifier SV 100 were used.

Participants

18 volunteers participated in the experiment. The sample was heterogeneous comprising one professional composer, three professional musicians, nine piano students and five non-musicians (including one dancer). The age ranged from 8 years to

61.

Stimuli

All stimuli were harmonic complex tones with a fundamental frequency of 450 Hz. The duration of the stimuli was varied between 9 notes/sec. and 0.5 notes/sec. The silent gap between two consecutive sound stimuli within a chain was half the duration of the preceding sound stimulus.

Six trials investigated tempo change. All of these stimuli consisted of six chronota (durations) but with different time ratios (1:2, 7:3 and 9:3). In three of these trials the tempo was changed by a factor 2 and in the other three trials by a factor 4.

In nine trials the length of the chain (between one and three beats) and the split ratio (1:1, 7:3 and 9:3) were varied simultaneously. For instance in one trial three chronota of equal length were compared to 6 chronota with the split ratio 9:1:9:1:9:1.

Four of the previous trials were reversed in another four trials (e.g. a chain of 9:1:9:1:9:1 was reversed to 1:9:1:9:1:9).

Finally, five trials compared simple chronotonic patterns with complex patterns (e.g. 1/4 - dotted1/2 compared to 1/6th - 1/16th - 1/4 - 1/16 - 1/16 - 1/8 - 1/8).

Procedure

Listeners rated the similarity on a 1 to 9 point scale. No guidance was given as regards to how to understand similarity. No practice trials were included. The experiment consisted of 24 trials (resulting in 48 when played in order *a* - *b* and order *b* -*a*). The order of each trial was randomized for each participant.

Results

Order Effects: At a significance level of 95% there was no order effect, except one trial. Shifting the significance level to 96% rendered this observation insignificant.

Tempo change: The three trials were the tempo changed by a factor 2 showed no significant difference with $F(5,18) = 1.7$, $p > 0.6$ nor where the factor was 4 with $F(5,18) = 2.43$, $p > 0.06$. However, the tempo change appeared to be significant with $F(11,18) = 3.51$, $p < 0.001$. Inputting these data into our model, we obtained a correlation of $r^2 = 0.74$ ($p < 0.02$) for $k_1 = 0.15$.

Split ratio: In trials where the length of the stimuli and the split ratio were varied simultaneously, we obtained a multiple correlation of $r^2 = 77$, p(split ratio) < 0.001 and p(length) < 0.02. Splitting one chronoton (duration) into two with a ratio of 9:1 (comparing one chronoton to two chronota with ratio 9:1) produces significantly higher correlation than splitting it into a ratio of 1:1(comparing a quarter note to two eighth notes).

Reversal: Comparing trials where a pattern was reversed a t-test ($p < 0.5$) revealed that reversion of the pattern has no effect on the similarity ratings.

Inputting all data (except tempo change) into the second predictor, we obtain a maximum correlation of $r^2 = 0.79$ ($p < 0.001$) for $k_2 =$ 1.28.

Conclusion

The experiment produced data supporting the hypothesis that the predictors as developed above are significant similarity predictors. Moreover, trials which could have produced conflicting data (such as reversal) failed to do so (with the exception of one trial).

6. GENERAL DISCUSSION

This paper set out to introduce a novel representation of chronotonic chains (sequence of musical durations) in form of atomic beats. Such representation allowed for the transformation of a given chain into any other chain of the same length via two reflections. This produced a reflection chain, which we found to be a similarity predictor (the closer the reflection line to the 0-axis the higher the similarity). Chains of different length were equalised via a factor. This factor acted as the second predictor. Both predictors served as the framework for an algorithm, which was subsequently put to its test in an experiment. It was found that not only did we obtain high correlations, but all trials (except one) were in agreement with the fundamental assumptions of the similarity model.

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