

# Dynamic Programming

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MUMT 611 - February 1, 2006

# Agenda

- An introduction
- Some sample problems
- How it may be used

# An Introduction

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	Flight-Number	From
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13:55	DI 7068	Köln Bonn
15:30	C9 1521	Herringsdorf
15:30	LH 9511	Herringsdorf
16:35	SN 2585	Brüssel
17:50	HE 518	Dortmund
17:55	DI 7072	Köln Bonn
18:10	D5 116	Dortmund
18:15	3L 306	Friedrichshafen
19:00	C9 1586	Saarbrücken

# Takes big problems...



breaks them into smaller problems

$$((2 \times 5) \times 4)$$

+

$$((2 \times 5) \times 2)$$

solves & caches smaller common results  
(which are easier to compute)

$$(2 \times 5 = 10) \rightarrow m$$

$m \times 4$

+

$m \times 2$

and solves the larger problem.

60

That was easy!  
Try this...

calculate fib(5)

0  
I

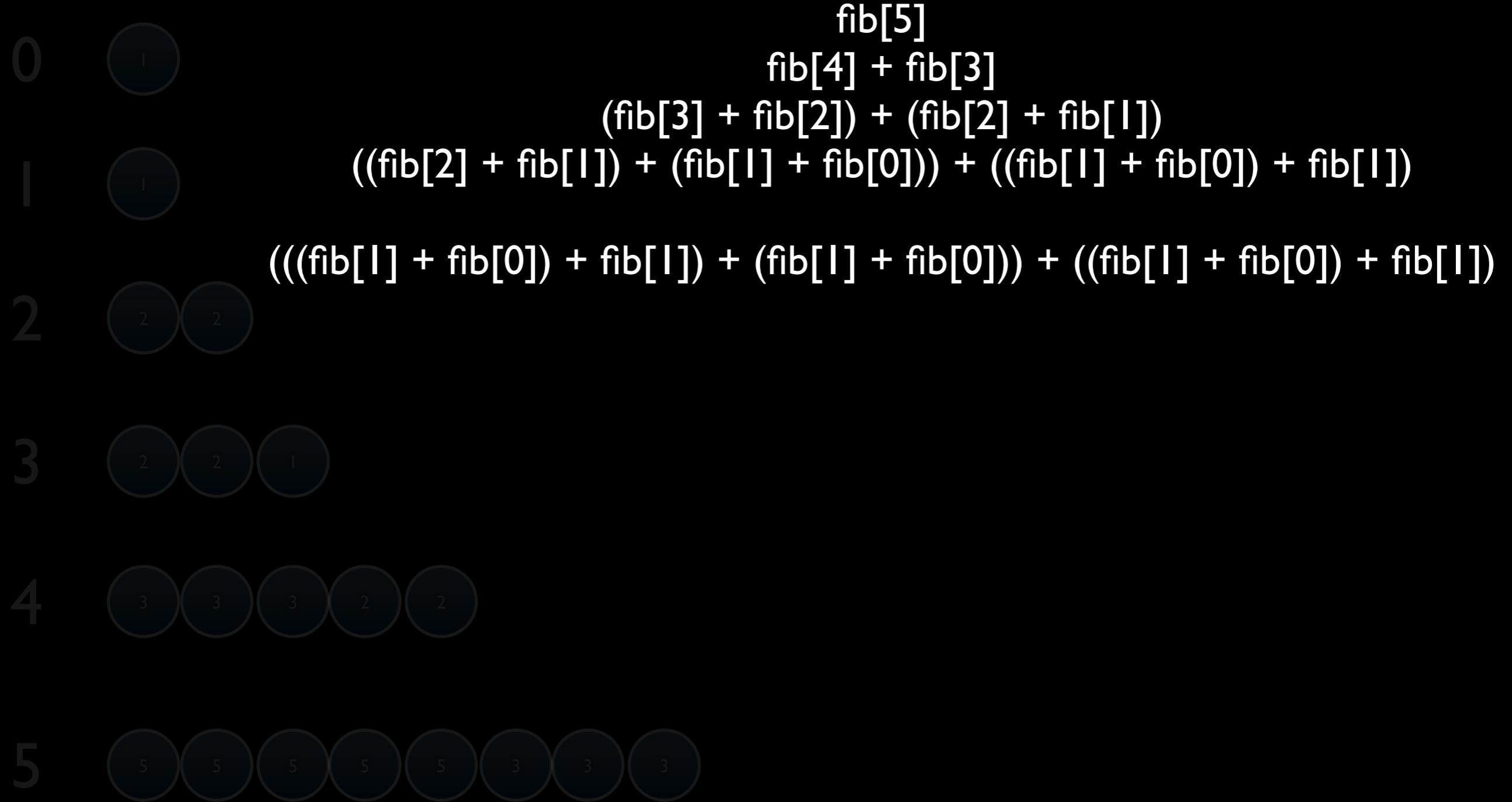
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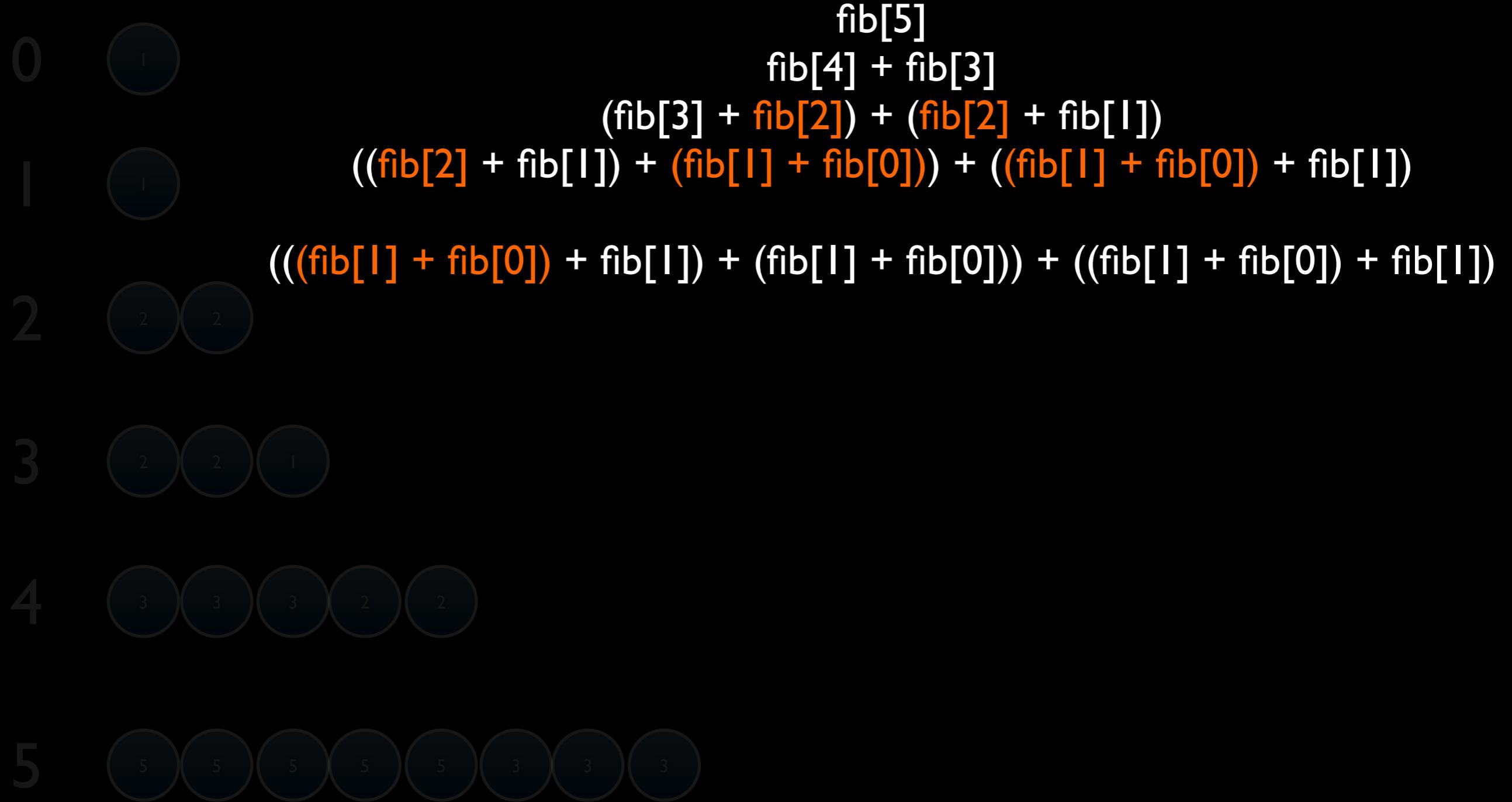
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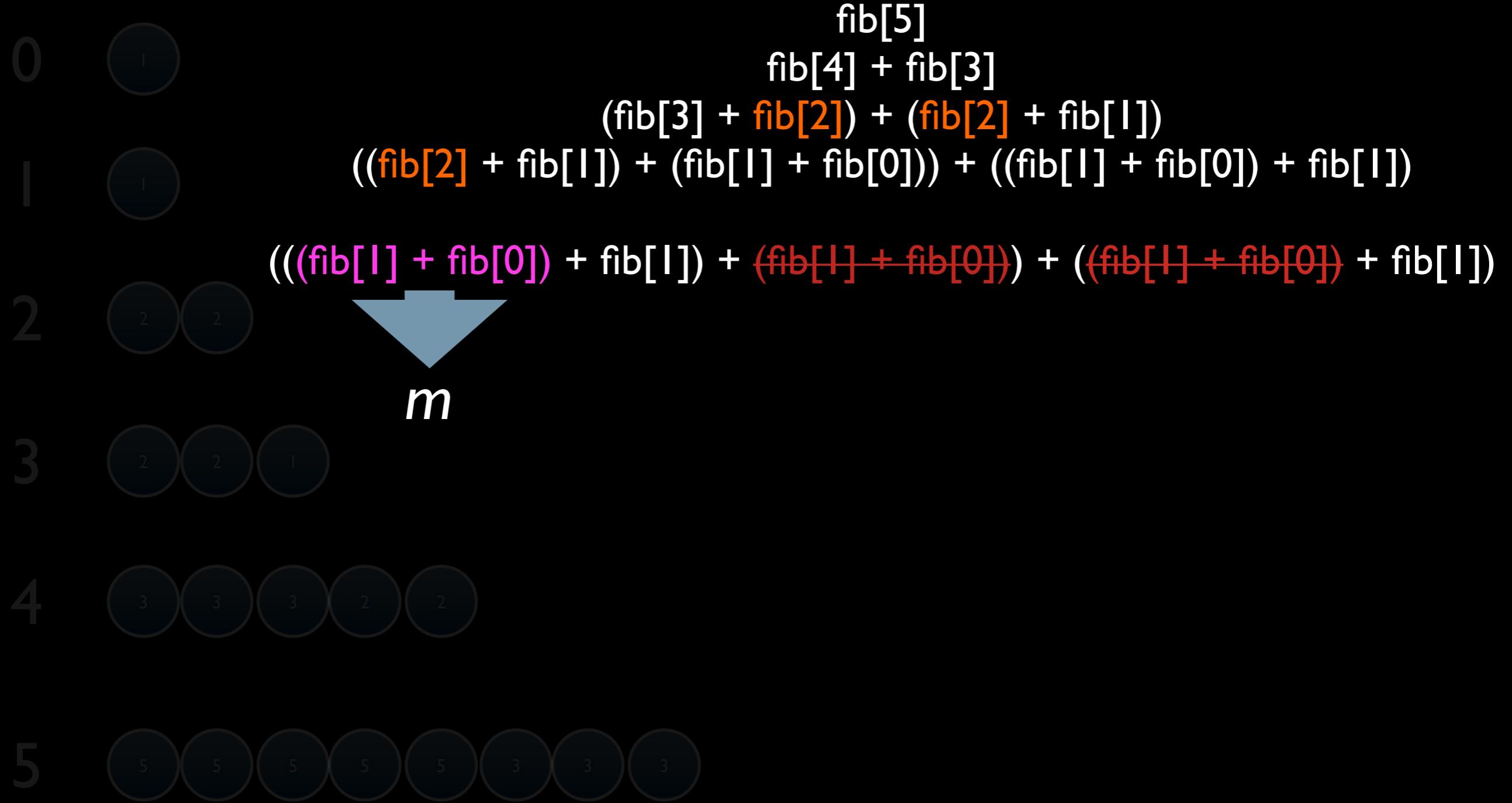
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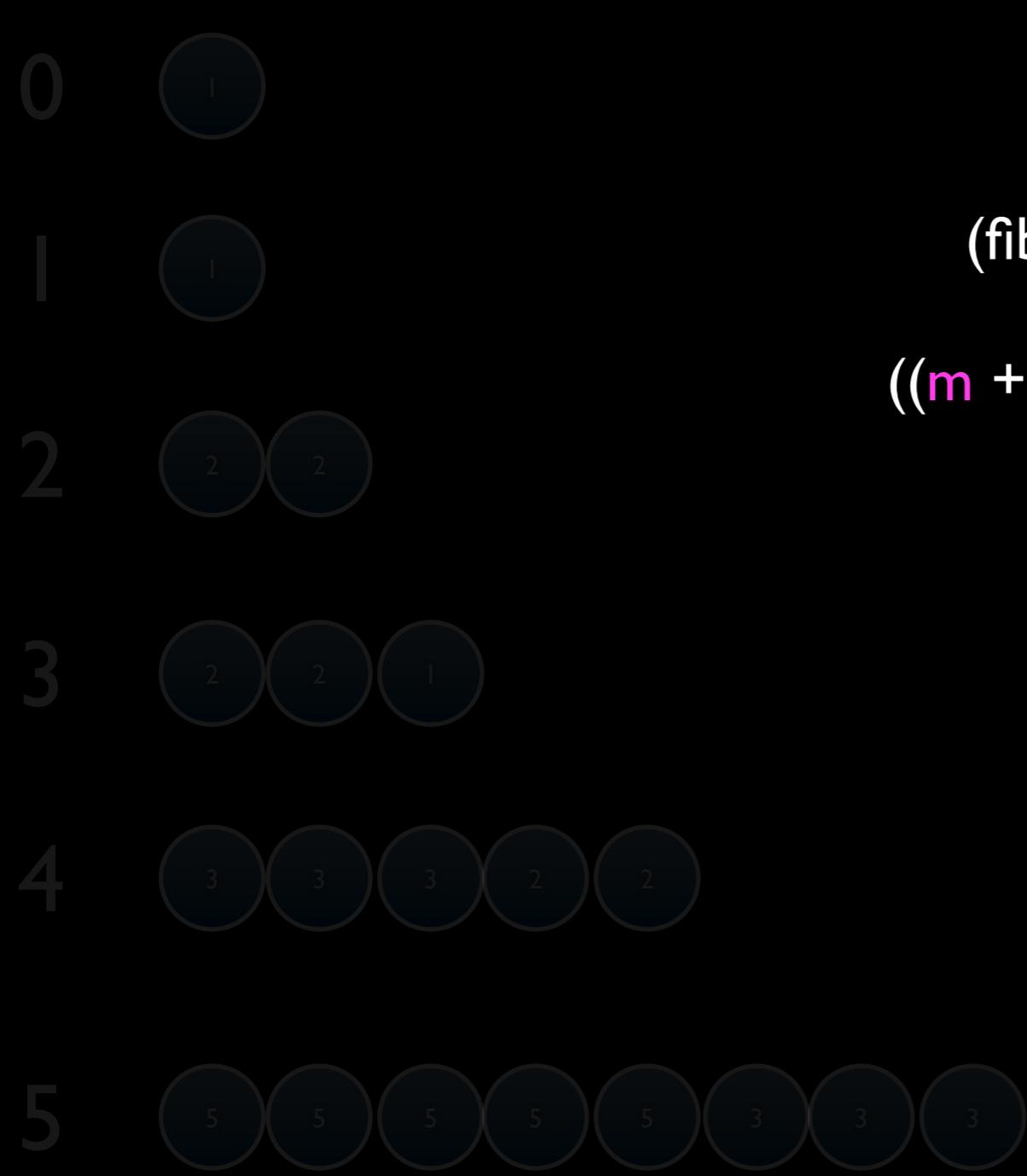
4  
3  
3  
3  
2  
2

5  
5  
5  
5  
5  
3  
3  
3









$\text{fib}[5]$   
 $\text{fib}[4] + \text{fib}[3]$

$(\text{fib}[3] + m) + (m + \text{fib}[1])$

$((m + \text{fib}[1]) + m) + (m + \text{fib}[1])$

# Dynamic Programming

Optimal  
Substructure

Best possible  
solution to  
subproblems  
(lowest ‘cost’)

# Dynamic Programming

Optimal  
Substructure

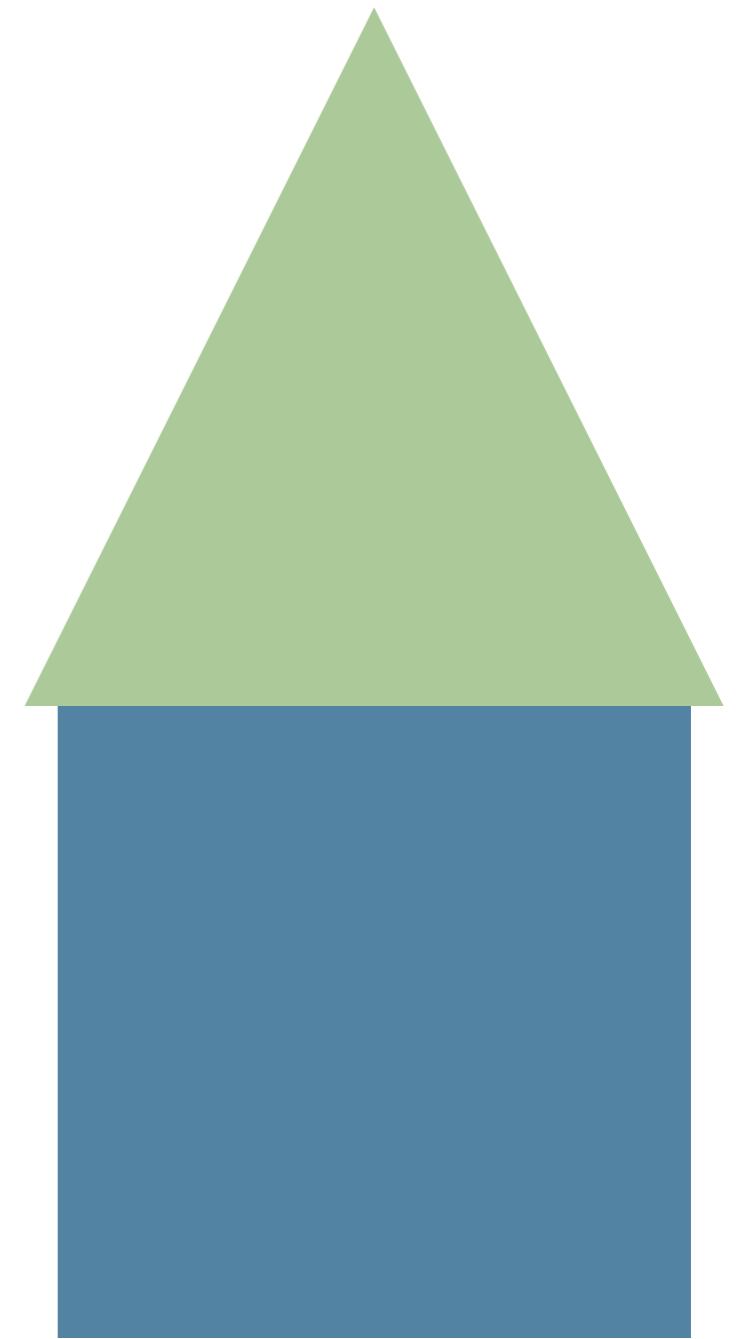
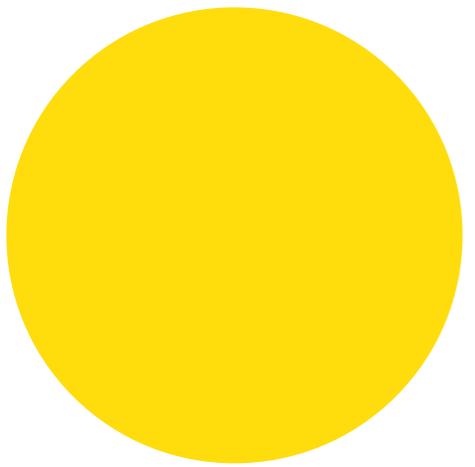
Overlapping  
Subproblems

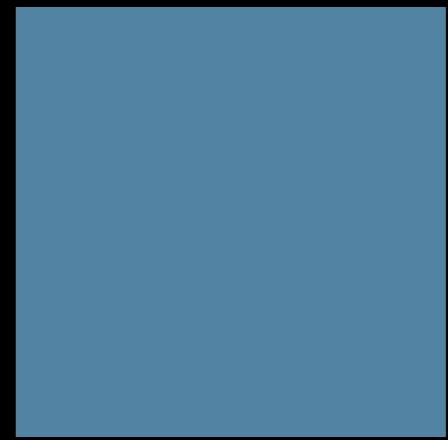
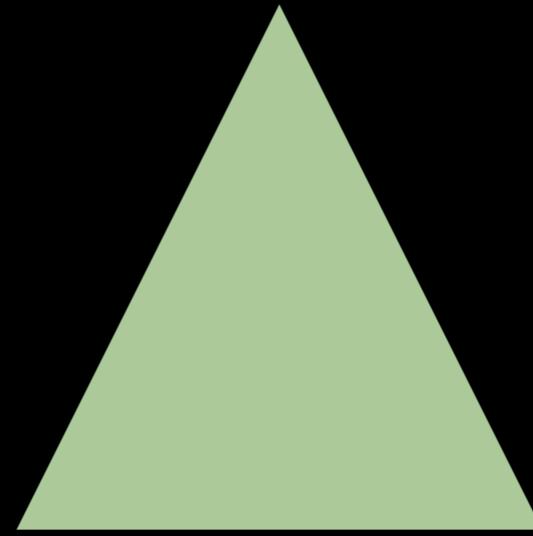
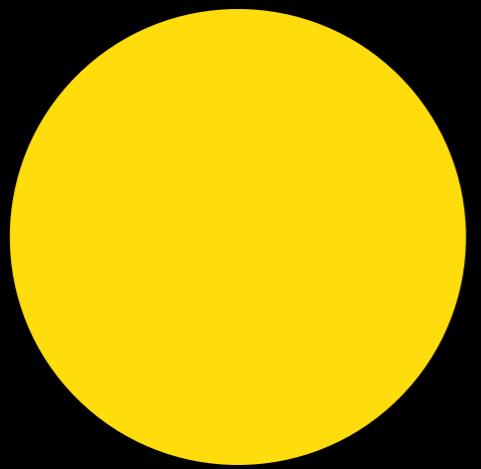
Best possible  
solution to  
subproblems  
(lowest ‘cost’)

Each subproblem  
has components  
whose solutions are  
the same

# Applications

# Computer Recognition

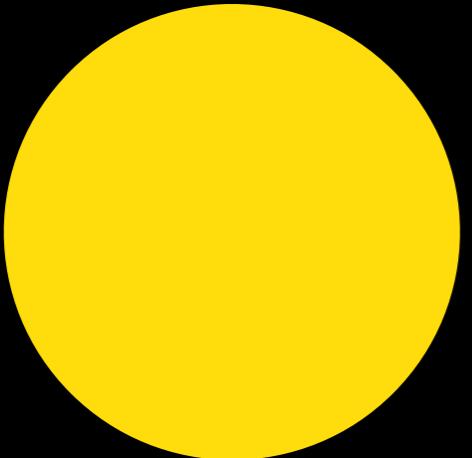
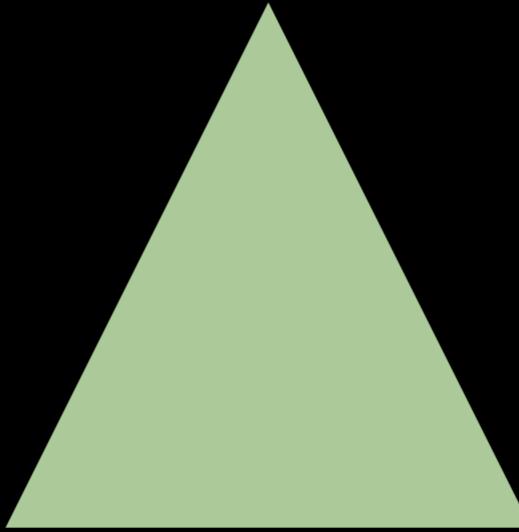
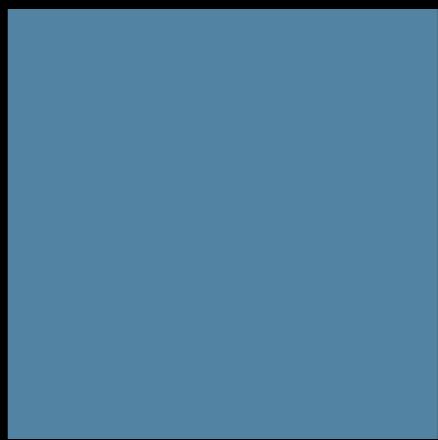


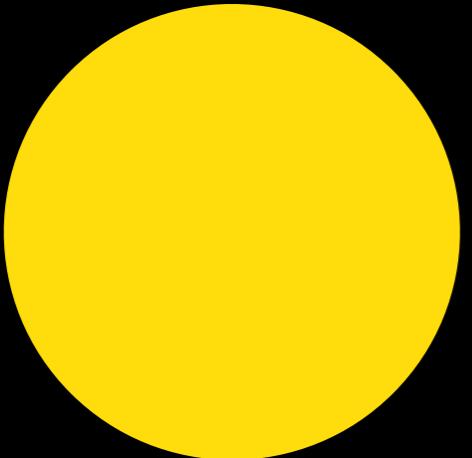
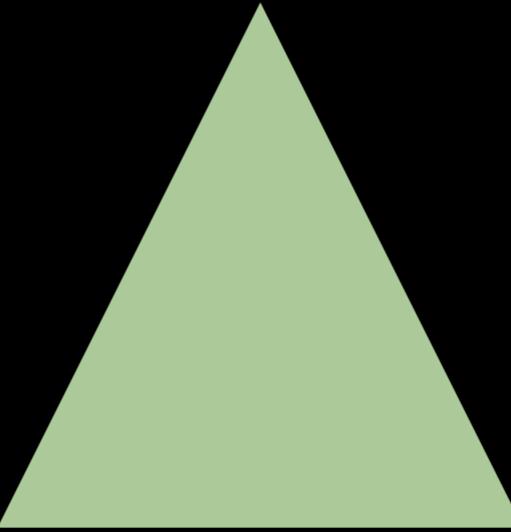
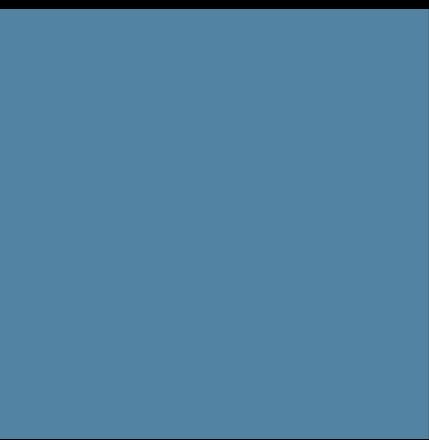


isRound  
isYellow  
isHigh

isTriangle  
isGreen  
isOnTopOfSquare

isSquare  
isBlue  
isBelowTriangle

								
<p><b>isRound</b>  <b>isYellow</b>  <b>isHigh</b></p>	<p><b>isTriangle</b>  <b>isGreen</b>  <b>isOnTopOfSquare</b></p>	<p><b>isSquare</b>  <b>isBlue</b>  <b>isBelowTriangle</b></p>						
<b>isRound</b>	<b>isYellow</b>	<b>isHigh</b>	<b>isTriangle</b>	<b>isGreen</b>	<b>isOnTopOfSquare</b>	<b>isSquare</b>	<b>isBlue</b>	<b>isBelowTriangle</b>
compute Shape	compute Colour	compute Position	compute Shape	compute Colour	compute Position	compute Shape	compute Colour	compute Position

								
<p><b>isRound</b>  <b>isYellow</b>  <b>isHigh</b></p>	<p><b>isTriangle</b>  <b>isGreen</b>  <b>isOnTopOfSquare</b></p>	<p><b>isSquare</b>  <b>isBlue</b>  <b>isBelowTriangle</b></p>						
<b>isRound</b>	<b>isYellow</b>	<b>isHigh</b>	<b>isTriangle</b>	<b>isGreen</b>	<b>isOnTopOfSquare</b>	<b>isSquare</b>	<b>isBlue</b>	<b>isBelowTriangle</b>
compute Shape	compute Colour	compute Position	compute Shape	compute Colour	compute Position	compute Shape	compute Colour	compute Position

Look for similar  
solutions to different  
problems

Look for the optimal  
solution to every  
problem

# Comments?