



EE 5345

Biomedical Instrumentation

Lecture 22: slides 417-427

Carlos E. Davila, Electrical Engineering Dept.
Southern Methodist University

slides can be viewed at:

[http:// www.seas.smu.edu/~cd/ee5345.html](http://www.seas.smu.edu/~cd/ee5345.html)



The Viterbi Algorithm

- Initialization ($t = 0$):

$$\mathbf{d}_0(i) = \mathbf{p}_i b_i(O_0), \quad 0 \leq i \leq N - 1$$

$$\Psi_1(i) = 0$$

- Time Recursion

For $1 \leq t \leq T-1$, $0 \leq j \leq N-1$

$$\mathbf{d}_t(j) = \max_{0 \leq i \leq N-1} [\mathbf{d}_{t-1}(i) a_{ij}] b_j(O_t)$$

$$\Psi_t(j) = \arg \max_{0 \leq i \leq N-1} [\mathbf{d}_{t-1}(i) a_{ij}]$$



The Viterbi Algorithm (cont.)

- Termination:

$$P_{\max} = \max_{0 \leq i \leq N-1} [\mathbf{d}_{T-1}(i)]$$

$$i_{T-1} = \arg \max_{0 \leq i \leq N-1} [\mathbf{d}_{T-1}(i)]$$

- State sequence backtracking:

For $t=T-2, T-3, \dots, 0$

$$i_t = \Psi_{t+1}(i_{t+1})$$



Backward Variable

$$\mathbf{b}_t(i) = \Pr(O_{t+1}, O_{t+2}, \dots, O_{T-1} | i_t = Q_i, \mathbf{I})$$

To understand this variable, assume that the current time step is “ t ”, the current state is “ Q_i ”, and we know the probabilities:

$$\mathbf{b}_{t+1}(j), \quad j = 0, \dots, N - 1$$

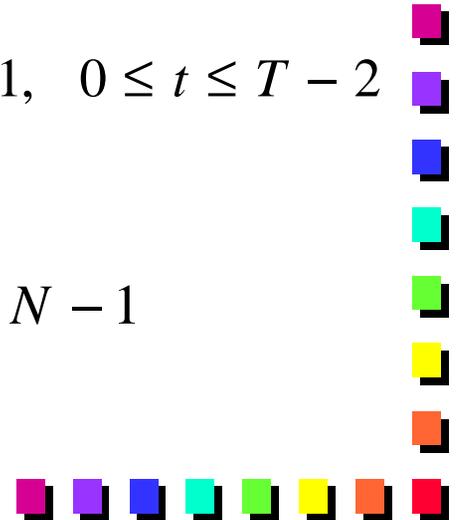
then it should be clear that:

$$\mathbf{b}_t(i) = \sum_{j=0}^{N-1} a_{ij} b_j(O_{t+1}) \mathbf{b}_{t+1}(j), \quad 0 \leq i \leq N - 1, \quad 0 \leq t \leq T - 2$$

since each of the N events:

$$O_{t+1}, O_{t+2}, \dots, O_{T-1} | i_t = Q_j, \quad j = 0, \dots, N - 1$$

are disjoint.



Backward Variable (cont.)

The backward variable can be computed recursively, moving backward in time.

1. initialize at $t = T - 1$,

$$\mathbf{b}_{T-1}(i) = 1, \quad i = 0, \dots, N - 1$$

2. for $t = T - 2 : -1 : 0$

$$\mathbf{b}_t(i) = \sum_{j=0}^{N-1} a_{ij} b_j(O_{t+1}) \mathbf{b}_{t+1}(j), \quad 0 \leq i \leq N - 1$$



More Definitions

The probability of landing in state Q_i at time t , given the observation sequence O is:

$$\mathbf{g}_t(i) \equiv \Pr(i_t = Q_i | O, \mathbf{I})$$

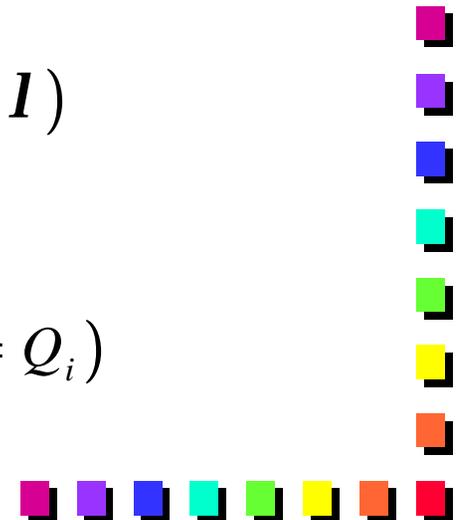
consider the previous definitions:

$$\mathbf{a}_t(i) = \Pr(O_0, O_1, \dots, O_t, i_t = Q_i | \mathbf{I})$$

$$\mathbf{b}_t(i) = \Pr(O_{t+1}, O_{t+2}, \dots, O_{T-1} | i_t = Q_i, \mathbf{I})$$

hence, for a given model λ :

$$\mathbf{a}_t(i)\mathbf{b}_t(i) = \Pr(O_0, O_1, \dots, O_{T-1} \cap i_t = Q_i)$$



More Definitions (cont.)

Hence:
$$\mathbf{g}_t(i) = \frac{\mathbf{a}_t(i)\mathbf{b}_t(i)}{\Pr(O|I)}$$

now consider the probability that we go from state Q_i at time t to state Q_j at time $t+1$ given the observation O :

$$\xi_t(i, j) \equiv \Pr(i_t = Q_i, i_{t+1} = Q_j | O, \lambda)$$

it follows that

$$\mathbf{x}_t(i, j) = \frac{\mathbf{a}_t(i)a_{ij}b_j(O_{t+1})\mathbf{b}_{t+1}(j)}{\Pr(O|I)}$$



More Definitions (cont.)

the average number of transitions made from Q_i :

$$\sum_{t=0}^{T-2} \mathbf{g}_t(i)$$

the average number of transitions made from Q_i to Q_j :

$$\sum_{t=0}^{T-2} \mathbf{x}_t(i, j)$$



Solution to Problem 3: Baum-Welch Algorithm

0. Initialize A , B , and Π

1. Compute $\alpha_t(i)$, $\beta_t(i)$ and $\Pr(O|I)$

2. Compute $\mathbf{x}_t(i, j)$ and $\mathbf{g}_t(i)$

$$\mathbf{x}_t(i, j) = \frac{\alpha_t(i) a_{ij} b_j(O_{t+1}) \beta_{t+1}(j)}{\Pr(O|I)} \quad \left(\begin{array}{c} \alpha_t(i) \\ \beta_{t+1}(j) \end{array} \right)$$

3. Compute $\mathbf{p}_i = \mathbf{g}(i), \quad i \leq N - 1$

4. Compute
$$\frac{\sum_{t=0}^{T-2} \mathbf{x}_t(i, j)}{T-2} \mathbf{g}_t(i)$$



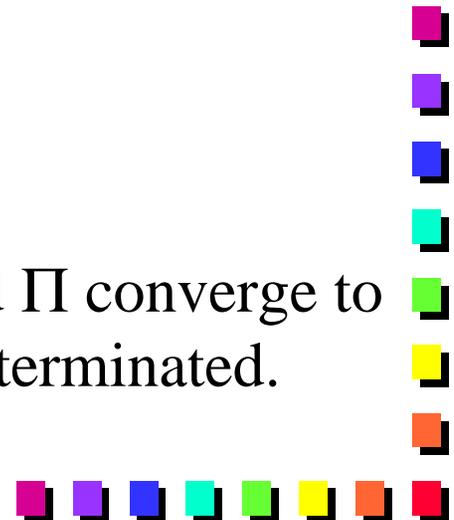
Baum-Welch Algorithm (cont.)

5. Compute

$$b_j(k) = \frac{\sum_{t=0}^{T-1} g_t(j)}{\sum_{t=0}^{T-1} g_t(j)}$$

7. go to step 2

$\Pr(O|I)$ should continue to increase until A, B, and Π converge to optimum values, at which point the algorithm is terminated.



Case Study: Coast et al.

- Used continuous density for observations:

$$b_i(v) = \frac{1}{\sqrt{2\pi s_i}} e^{-0.5((v-\mu)/s_i)^2}$$

This alters most of the formulas we looked at but the basic ideas remain the same.

- Observations consisted of actual ECG samples.
- Used several rhythm HMM models in parallel
- Viterbi algorithm was used to select the most likely sequence (and hence rhythm type).

