On the Bore Shape of Conical Instruments

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1. INTRODUCTION

Woodwind music instruments are either based on a cylindrical or a conical air-column. To behave properly, any woodwind instrument must be so designed that the lowest resonances of each note of the first register are well aligned, i.e. that they are harmonic [1]. This is necessary for two reasons. First, this ensures a proper collaboration of each resonance in creating a rich, stable tone in the first register. Second, this ensures that the second register is well tuned with the first.

Conical-bored instruments such as the saxophone deviate in geometry from a perfect cone at both extremities. At the large end, the flare angle increases and forms the bell, enhancing the radiation of sound. The small end of the air column is normally cylindrical to allow a mouthpiece to be inserted into or over it. For saxophones, the larger end of the neck is also cylindrical so that it can be inserted into the main body of the instrument. Do these deviations hinder or improve the harmonicity of the instrument?

In this paper, we study the impact of the bore shape on the harmonicity of the resonances for each note of simplified saxophone-like instruments. A number of bore shapes were investigated including cones of varying taper angle and upper bore deviations in order to gather insight into what deviations might be necessary for the proper functioning of saxophones. The idea involves calculating the positions of the toneholes on a given bore shape for the fundamental resonance to correspond with the notes of the musical scale and then to calculate the deviation of the second resonance from perfect harmonicity. This is done with an optimization algorithm [2,3]. For a well-designed instrument, the second resonance should be within about 10 cents of two times the fundamental resonance frequency, where the interval in cents is calculated as $1200\log\left(f_2/f_1\right)$. One semitone equals 100 cents.

The transmission matrix method (TMM) provides an efficient means for calculating the input impedance of a hypothetical air column [4,5]. With the TMM, a geometrical structure is approximated by a sequence of one-dimensional segments, such as cylinders, cones, and closed or open toneholes, and each segment is represented by a matrix (TM) that relates its input to output quantities of pressure (P) and volume velocity (U). The multiplication of these matrices yields a single matrix, which must then be multiplied by an appropriate radiation impedance at its output. That is:

$$\begin{bmatrix} P_{in} \\ Z_0 U_{in} \end{bmatrix} = \left(\prod_{i=1}^{n} T_i \right) \begin{bmatrix} \overline{Z}_{rad} \\ 1 \end{bmatrix}. \qquad (1)$$

The normalized input impedance is then calculated as $\overline{Z}_{in} = P_{in} / Z_0 U_{in}$.

2. RESULTS

Many bore shapes were simulated. Four of them were selected for this paper. All geometries are made of a cylindrical mouthpiece of 15.8 mm diameter and 50 mm length followed by the air-columns. Approximating the mouthpiece as a cylinder may not be the most appropriate model but this is standard practice in the literature and no better geometry appears obvious. For the sake of simplicity, the cylindrical model was adopted. All four air-columns start with a diameter of 12.5 mm, as is typical for an alto saxophone (there is a diameter jump). The first air-column (A) is a straight conical bore with an angle of 3 degrees. The second air-column (B) is also a straight conical bore but with an angle of 3.5 degrees. The third air column (C) starts with a segment of cylindrical pipe of 25 mm length followed by a conical bore with an angle of 3 degrees. Finally, the last air-column (D) starts with a segment of conical bore of 50 mm length with an angle of 3.5 degrees followed by a conical bore with an angle of 3 degrees. The first 200 mm of these geometries are displayed in Fig. 1. The first register of the instruments is in tune within 1 cent.

An instrument with a larger angle has a smaller truncation ratio [6] and is expected to display better harmonicity. Similarly, an instrument with an increased taper in the upper part of the bore should also have a better harmonicity [7]. The geometries were selected to verify these hypotheses.

Figure 2 displays the deviation in cents of the second resonance for each note of the first register of the instruments. None of these instruments have sufficiently good harmonicity for all notes. For all geometries, the harmonicity is increasingly problematic for higher notes. Geometry A gives a decent harmonicity for the first 9 notes, after which the second resonance becomes increasingly too high. Geometry B, which differs from A only in the angle of

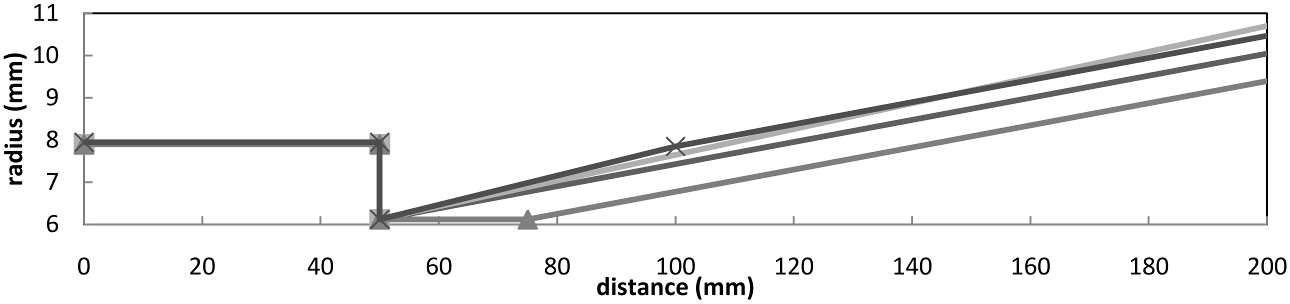

Figure 1 Radius (in mm) of bore of the top part of the four instruments: A (diamonds), B (squares), C (triangles) and D (crosses)

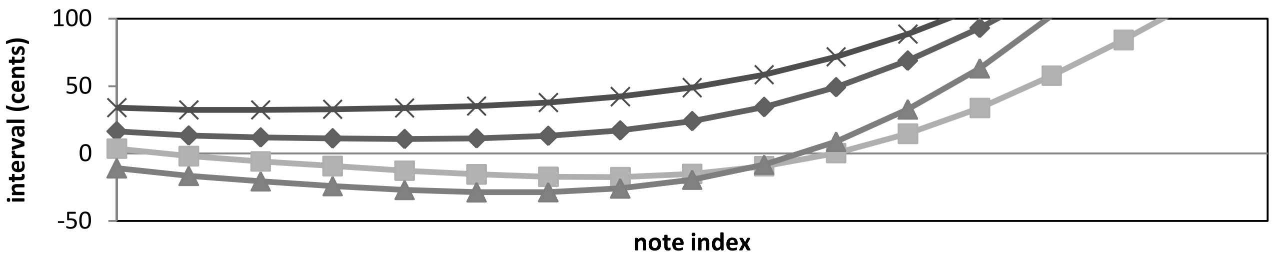

Figure 2 Frequency deviation in cents of the second resonance relative to twice the fundamental resonance for the four conical air-columns: A (diamonds), B (squares), C (triangles) and D (crosses).

conicity results in lower second resonance for all notes, which improves the harmonicity for many notes of the instruments but worsens other notes. For geometry C and D, the harmonicity is worse than for the straight cones.

In a recent paper [8], the input impedance of three complete saxophones was measured from which the resonance frequencies were estimated. These measurements correlated well with the calculated values based on their measured geometries and the harmonicity was very good compared to values of the current paper. This suggests that the deviations from a cone found on these instruments play a role in the proper harmonicity of the instrument and that simple modifications such as those studied in this paper are not sufficient. The question remains what bore shape might achieve the best harmonicity for each note.

3. CONCLUSIONS

It is generally admitted that the air-column shape of a saxophone is a cone, but this is only an idealized description. This brief study confirms that a straight conical tube is not an appropriate geometry for a saxophone, that deviations are necessary to bring the second resonance in harmonic relation with the first and that simple modifications in the upper part such as an increased angle or a segment of cylinder are not sufficient.

The application of an optimization algorithm that allows simultaneous variation of tonehole positions and bore shape could potentially lead to an improved design and a better understanding of the relation between the geometry of the bore and the quality of the instruments. Ongoing efforts on this topic are providing promising results.

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