A Comparison of Impedance Measurements Using One and Two Microphones

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Abstract:

Measurements of acoustic input impedance of wind instruments using two different approaches are presented. In the first approach, a tube is connected to the instrument and excited with broad-band noise. Signals recorded at microphone pairs placed along the tube are then analyzed to estimate the instrument input impedance. A calibration step is described, wherein the position of each microphone pair is determined from the measurement of a rigid termination. The second technique makes use of a long tube with a single microphone located at its midpoint. Using a swept sinusoid stimulus, the impulse response is measured for the tube, first with a rigid termination, and then with the system to be characterized attached. The system reflectance, and therefore its impedance, is found by comparing the first reflection from the tube end for both measurements. The design of the impedance probes and the data sampling and analysis procedures are presented. Measurements obtained using the two techniques are compared for various acoustic systems, including alto saxophones and fabricated conical objects.

INTRODUCTION

The measurement of acoustic impedance has been the subject of much research since the beginning of the last century and a great number of publications have been written on the subject. The reader interested to learn more about the historical origins and development of these techniques can read Benade and Ibis [1987] and Dalmont [2001]. Since the 1980's, with the development of signal processing tools and computers, two measurement techniques have become widely used: the two-microphone transfer function (TMTF) technique and pulse reflectometry.

The use of two microphones located in an acoustic transmission line to evaluate the impedance of an object date back to the early 19th century Beranek [see 1988]. The two-microphone transfer function technique, introduced by Seybert and Ross [1977], improved the previous one by the use of a broadband signal and Fourier analysis to evaluate impedance for the whole spectrum in one measurement. It has also been described by Chung and Blaser [1980b,a] and extended by Chu [1986] to include attenuation.

Pulse reflectometry originated from geophysical studies of the earth's crust but, throughout the 1970s and 1980s, it has been applied to the study of the vocal track [see Fredberg et al., 1980] and to musical instruments. The time-delay spectromety (TDS) technique reported here is based on the same principle as pulse reflectometry but achieves an improved signal to noise ratio by using wide-band signals of long duration, such as swept sines. That approach has never been reported before.

In the context of musical acoustic, we are mainly interested in the magnitude and frequency of the maxima and minima of strongly resonant bodies, which are generally more difficult to evaluate with accuracy.

The objective of this paper is to compare impedance measurements obtained with both techniques in order to identify and characterize possible discrepencies between the two, as well as to better assess the accuracy of the results and the importance of measurement errors. The same objects will be measured with two fundamentally different techniques and, if the measurements are accurate, their impedances should be identical inside the confidence interval [??].

We first detail the experimental setup, calibration procedures, and signal analysis methods for both techniques. We then present impedance measurement results for three objects: an alto saxophone neck, a short carbon fiber cone, and a long carbon fiber cone coupled with the neck. We conclude with a comparison of the advantages and disadvantages of both techniques in the context of musical acoustics.

TWO-MICROPHONE TRANSFER FUNCTION TECHNIQUE

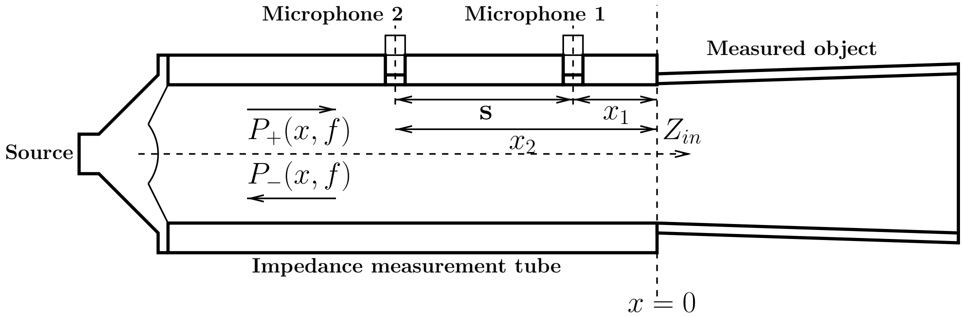

FIGURE 1. Diagram of the two-microphone measurement apparatus

In the two-microphone transfer function technique, the impedance of an object is evaluated from the measurement of the transfer function between two microphones located at different positions along a waveguide connected to that object. A horn driver emits a broad-band signal, such as white noise, in the waveguide over a time duration adequate to reduce variance in the results, as computed with a modified average periodogram.

This technique is based on the mathematical theory of one-dimensional planar pressure wave propagation in a cylindrical duct. Such waves, including attenuation, can be described by the equation

$$P(x,f) = P_+(x,f) + P_-(x,f) = Ae^{-\Gamma x} + Be^{\Gamma x}, \tag{1}$$

where A and B are the complex frequency-dependent amplitudes of the progressive and regressive traveling-wave components. The propagation parameter is defined as

$$\Gamma = \alpha + i\frac{\omega}{v_\phi}, \tag{2}$$

where α is the attenuation and v_ϕ the phase velocity. Estimation of this parameter has been described by Pierce [1989]. It can be approximate by $\Gamma = i\omega/c + (1+i)\alpha$, where $\alpha = \text{CST} \times \sqrt{f}$ and the constant depends on air properties.

From these equations and following a long mathematical development [Lefebvre, 2006], we can demonstrate that the impedance $\bar{Z}_{in}$ of an object located at $x = 0$ (see figure 1) is given by

$$\bar{Z}_{in} = \frac{Z}{Z_c} = \frac{H_{12}\sinh(\Gamma x_1) - \sinh(\Gamma x_2)}{H_{12}\cosh(\Gamma x_1) - \cosh(\Gamma x_2)}, \tag{3}$$

where H_{12} is the transfer function between the two microphones and Z_c is the characteristic impedance.

Microphone Pair	Distance	Frequency Range (Hz)
1 and 2	3 cm	575 - 4600
1 and 3	12 cm	290 - 1150
1 and 4	36 cm	95 - 380

Table 1. microphone pairs use in our measurement apparatus

This approach is based on one-dimensional wave propagation and thus, it is limited in frequency to the first higher-order mode that occurs at $f = 1.84c/(2\pi a)$, where a is the cylinder radius and c is the speed of sound. For our measurement system, the cutoff frequency is approximately 8 kHz ($a = 0.006$m).

This measurement technique is also incapable of providing results at critical frequencies where the pressure signals become linearly dependent, which corresponds to half-wavelengths that are an integer multiple of the microphone spacing:

$$f_c = mc/2s, m = 1, 2, ..., N. \tag{4}$$

The consequence is that we need several pairs of microphones to cover a sufficient frequency range for musical instrument characterization. To achieve a frequency range of 100 – 5000 Hz, we use four microphones. Table 1 indicates the microphone distances and valid frequency ranges. Final impedance results are realized by concatenating impedances from three microphone pairs.

Prior to the measurement, we perform a relative calibration of microphones pairs, as described by Seybert and Ross [1977] and Krishnappa [1981], in order to eliminate frequence response differences between them. This calibration is performed using a special apparatus where the four-microphnes are located at the same reference plane and exposed to a broadband noise signal. This operation is as simple as measuring an objects but requires the microphones to be relocated. The microphone positions in Eq. (3) can also be calibrated (fine tuned) with a measurement obtained when the plane at $x = 0$ is rigidly terminated, the transfer function between two microphones being theoretically known for such a boundary condition Lefebvre [see 2006]:

$$H_{12} = \frac{\cosh(\Gamma x_2)}{\cosh(\Gamma x_1)}. \tag{5}$$

The attenuation, which is higher than predicted if surface roughness increases, can be ajusted as well using the magnitude of those maxima and minima.

We evaluate the transfer function H_{12} between the recorded signals at the two microphones with the total least square formulation, which reduce the impact of noise [see P.R. White, 2006]:

$$H_{12} = C_{12} \times \frac{S_{p_2p_2} - S_{p_1p_1} + \sqrt{(S_{p_1p_1} - S_{p_2p_2})^2 + 4|S_{p_1p_2}|^2}}{2S_{p_2p_1}} \tag{6}$$

where $S_{p_1p_1}$ is the auto-correlated spectral density of first microphone's signal, $S_{p_1p_2}$ is the cross-correlated spectral density between microphone 1 and 2, etc. C_{12} is the calibration function previously measured.

TIME-DELAY SPECTROMETRY

The time-delay spectrometry (TDS) technique uses a setup with a single microphone and a calculation based on two measurements. The apparatus consists of a horn driver connected to a long probe tube and a microphone located at its mid-point, as illustrated in Fig. 2. After performing a measurement with the probe tube rigidly terminated, the object to be measured is attached to the end of the tube and another measurement is made. In contrast to traditional pulse reflectometry techniques, a long duration source signal, such as a swept sine, is used to make the measurements.

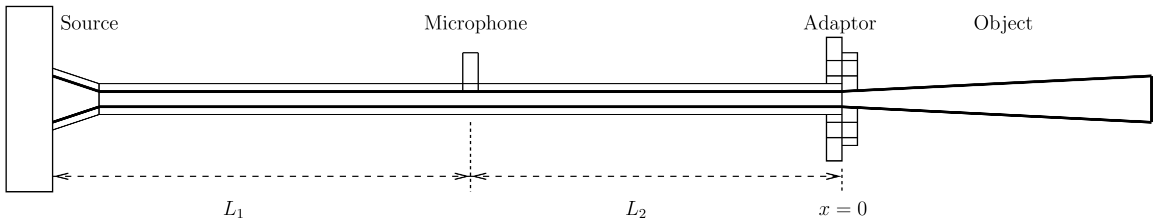

FIGURE 2. Setup for the one-microphone measurement system.

The impulse responses of the system fitted alternately with a rigid termination and with the object of interest are measured by deconvolving the recorded signal, $y(t)$, from the input or source signal $x(t)$:

$$ir(t) = \Re\left\{\text{IFFT}\left[\frac{\text{FFT}(y(t))}{\text{FFT}(x(t))}\right]\right\}. \tag{7}$$

Because the source signal $x(t)$ is non-zero at all frequencies of interest, there are no stability problems with this calculation. Each impulse response then consists of a series of pulses corresponding to an initial pulse (p_1) from the driver, its reflection from the reference plane at $x = 0$ (p_2), the reflection back from the driver (p_3), etc. The reflection coefficient $R_{in}(f)$ of the measured object is evaluated by taking the ratio of the Fourier transform of the time-windowed first reflection from the object, FFT(p_{2o}), and the Fourier transform of the time-windowed first reflection from the rigid termination, FFT(p_{2r}):

$$R_{in}(f) = \frac{\text{FFT}(p_{2o})}{\text{FFT}(p_{2r})}. \tag{8}$$

The input impedance of the object is then calculated as:

$$\bar{Z}_{in} = \frac{1 + R_{in}}{1 - R_{in}}. \tag{9}$$

It is necessary that the impulse response of the object to be measured be shorter than the corresponding propagation time along one length of the probe tube. Alternately, the impulse response must decay in time before its reflection from the driver returns to the microphone position. The consequence is that a longer measurement tube is needed to measure objects with long impulse responses. As has already been pointed out by Sharp [1996, pg. 84], the use of a longer tube implies more propagation losses and, because losses increase with frequency, a reduced frequency range. This study made use of two different probe tubes: a straight aluminum pipe of 5 meter length and 0.015 meter diameter (referred to as TDS-straight); and a coiled copper pipe of 18.29 meter length and 0.0127 meter diameter (referred to as TDS-coil).

RESULTS

[Add an explanation about the theory of the cone input impedance with losses...]

Measured objects are (1) an alto saxophone neck Selmer series II ($a_1 = 12.6$ mm, $a_2 = 22.7$ mm, $L = 195$ mm), (2) a short carbon fiber cone ($a_1 = 12.3$ mm, $a_2 = 33.2$ mm, $L = 40.3$ mm) and (3) a long carbon fiber cone ($a_1 = 23.5$ mm, $a_2 = 72.0$ mm, $L = 834$ mm) coupled with the neck.

The same horn driver (JBL 2426H) and acquisition card (RME Fireface 800) have been used with all the probes. Microphone capsules (Senheiser KE4-211) amplified with a circuit based on AD822A operational amplifier are used with the TMTF technique while a **** have been used with the TDS probes.

In order to compare impedance measurements of the same objects with different apparatus, they need to be done in the same atmospheric conditions for the speed of sound to be the same. If it is not the case, the resonances of the objects will be shifted in frequency. In order to normalize the results, we must scale the frequency axis by a factor proportional to the ratio of the speed of sound for the measurements to a reference speed of sound. The air temperature was $24.6°C$ with the TMTF measurements, $22.4°C$ with the TDS (coil) and $22.6°C$ with the TDS (tube). The frequency scaling factor is about 0.997 to normalize the TDS measurements to the TMTF one.

The coupling between the object under investigation and the measurement apparatus must be free of any discontinuities for the results to be free of systematic errors caused by the excitation of evanescent modes. As our three probes have slightly different input diameters, we observed that this error correspond to a positive shift of the impedance in the logarithmic scale. The shift is more important for the tube than for the coil as should be expected by the more important diameter mismatch. A method to correct the measurements have been presented by Maarten van Walstijn and Sharp [2005] from a theory developed by Pagneux et al. [1996] and is under investigation in our research group. A shift to the time-delay spectrometry measurements have been added and adjusted to fit the reults taken with the two-microphone probe as that one didn't present a diameter mismatch. The shift is -4.00 dB for the straight prove and -1.85 dB for the coiled one.

With the time-delay spectrometry technique, as have been explain in the theory, the high frequencies in the signal get attenuated in the long tube and reduce the signal to noise ratio. The result is a greater noise in the impedance measurement. This behavior can be observed in Fig. 3 for the coil where the noise begin to be noticeable at quite a low frequency (between 2kHz and 3kHz).

We can also observe on figure 5 that the measurement of the long cone do not work with the TDS (straight). In that case, the requirement that the impulse response of the object be small enough for the separation of the pulses break.

Though difficult to distinguisg in the figures, we noticed that the two-microphone transfer fonction technique present noisier results and that this noise is stronger at the maxima and minima of the impedance. Those extrema are smoother with the TDS technique and, especially with the coil, they are evaluated slightly stronger (about 2dB) and closer to the theoretical predictions.

We can also observe that the match between the impedance made with the three microphone pairs is quite good, which mean that the calibration procedure works well.

Although the TMTF technique gives quite good results and that the match between the concatenated impedance is nearly perfect, it stays tricky to use correctly. The major problem is that the system in itself get many resonances due to the relativly short length of the probe and to the reflectivity of the horn driver. This cause a relativly large difference in the amplitudes of the signal at resonances and anti-resonances of the system. When the signal get stronger, there is a risk of distorsion in the microphones while in the opposite situation, the S/N ratio get quite poor and reduce the quality of the results. The same problem appear with the calibration apparatus. For all those reasons, the gains of the microphones and the level of the speaker are difficult to set up. It is also necessary to take the measurements in a low noise environment while the TDS technique work pretty well in a computer lab environment.

The comparison of the results of the measurements with the theory shows interresting features. The best match arise for the straight carbon fiber cone (which is the closest to a real cone but still present a small curvature). For that object, the theory predict a slightly lower frequency for the first maxima (-17 cents) but a higher frequency for the third (+16 cents). It predicts lower frequencies for all the minima. A similar behavior arise for the neck which have an important curvature. The theory predict a lower first maxima (-64 cents) and a higher second (+36 cents) and third (+40 cents) maxima. On the contrary, the first minima is lower in theory (-10 cents) but the third is higher (+34 cents). The curvature might be the cause of those variations.

In the case of the long cones, both TMTF and TDS gives almost identical results while the theory is quite off. The first two maxima and minima magnitudes are much more important in theory (around 10 dB). They get closer for next extrema but minima stay lower. That difference is obvious on Fig. 5. Are those discrepancies due to the neck/cone jonction, the apparatus/object coupling, the neck curvature, the radiation approximation, ... ? We can at least tell that the measurements

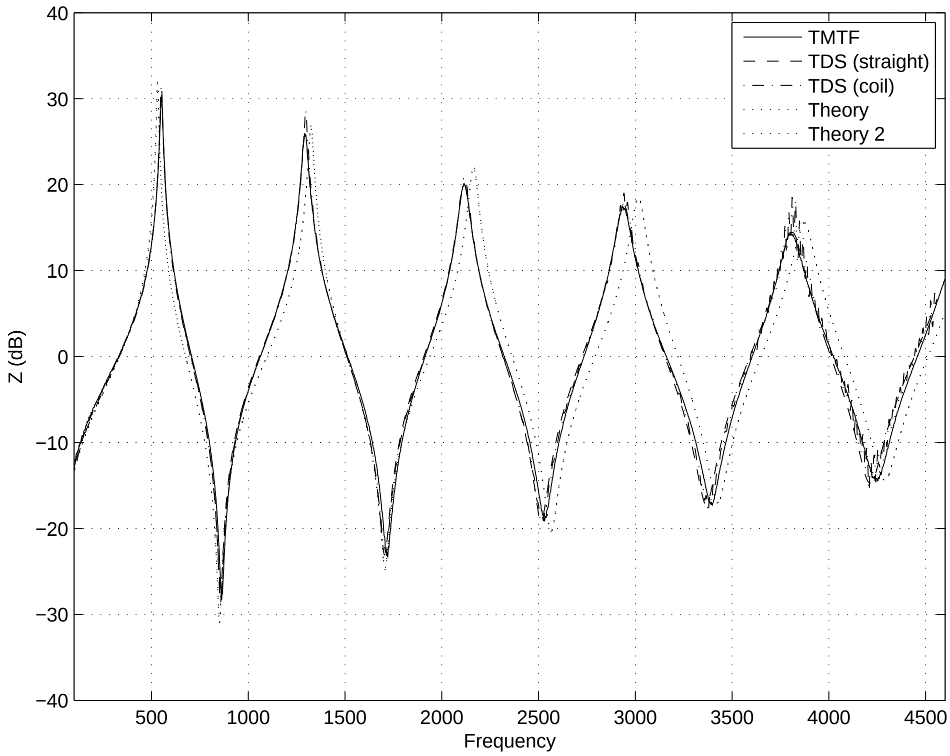

FIGURE 3. Magnitude and phase of the input impedance of the alto saxophone neck.

are representative of the true impedance of the system because the two techniques gives almost identical results.

CONCLUSIONS

In the process of taking measurements with both types of technique, we realize that TDS was much simpler to work with. The fact that there is only one microphone and no need for calibration makes it quite efficient. The problems we had with the resonances in the TMTF technique increases the time needed to correctly set up the system.

Advantaged of the TMTF technique is that the apparatus is compact and that we can measure high frequency impedance easily. On the other side, the need for many microphones add a calibration step which is not necessary for the TDS.

The TDS apparatus uses long tubes and is not easily portable. The most important problem of that technique is that the frequency range of the measurement is reduced if we need to measure object with long impulse response. There is a compromise between the largest possible object you can measure and the highest possible frequency.

Another important advantage of the TDS technique is that the maxima and minima are cleaner and, from the experiments we made, looks closer to the theoretical impedance.

To confirm our results we plan to perform new measurments with a TDS probe with the same inner diameter as the object under investigation.

Coupling must be as free as possible of discontinuities. For useful comparaison of measurements, they must have been measured at the same temperature or correctly normalized.

In order to improve the performance of the TMTF technique, we would like to develop a procedure that works with a chirp whose amplitude is controlled to counteract resonances of the system in the hope that it might reduce the noise in the results.

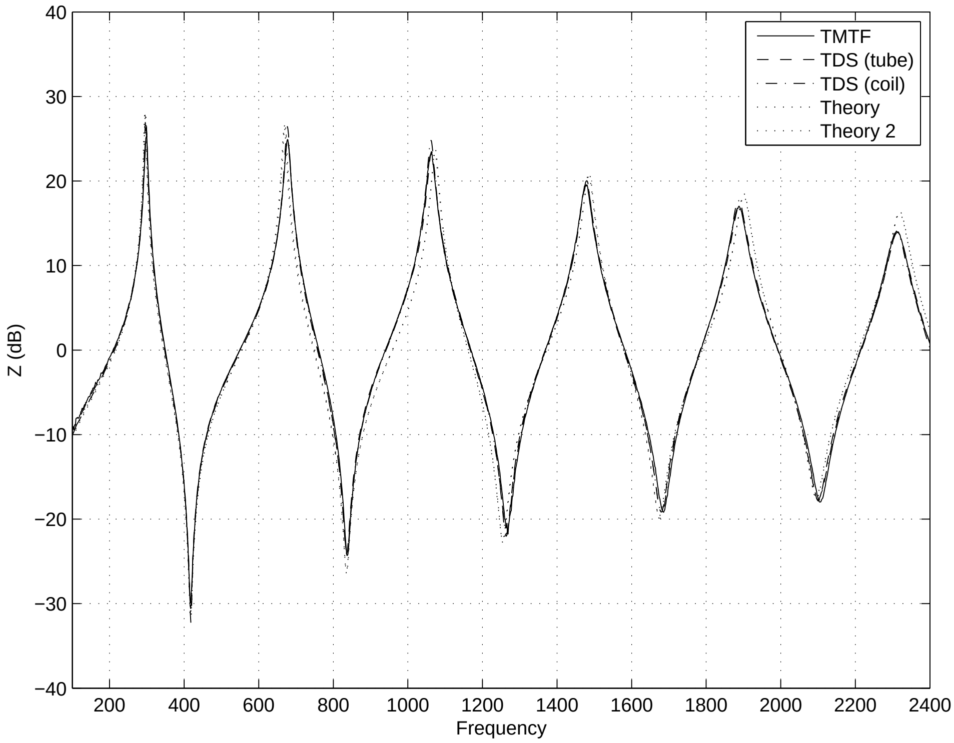

FIGURE 4. Magnitude and phase of the input impedance of the small carbon fiber cone.

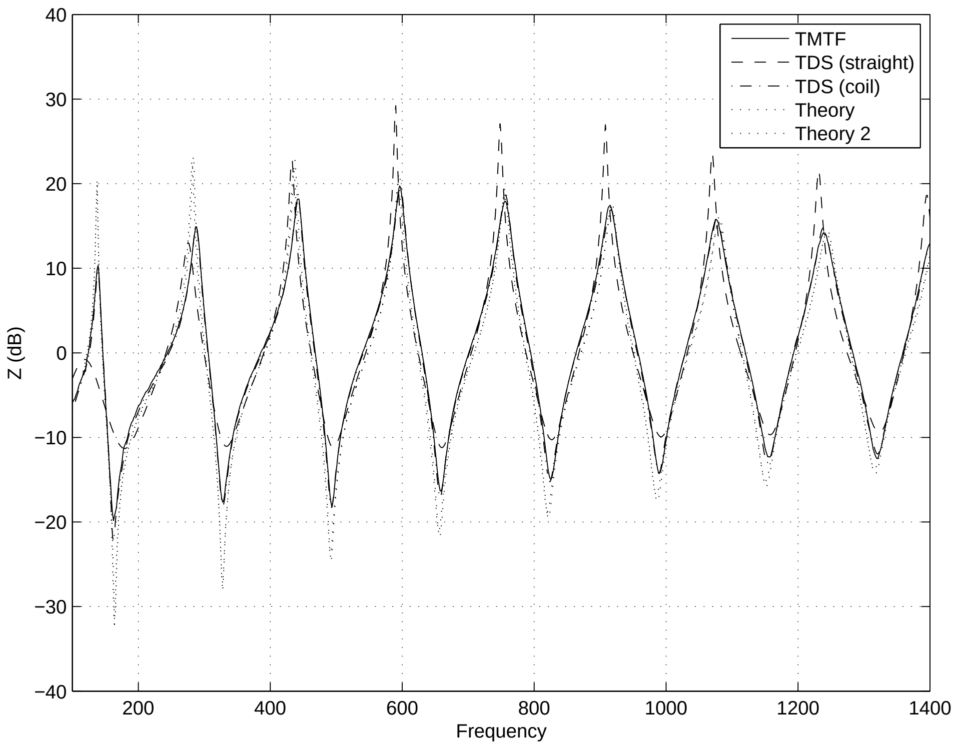

FIGURE 5. Magnitude and phase of the input impedance of the long carbon fiber cone.

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