Real-time Computer Modeling of Woodwind Instruments

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Abstract: This paper presents a digital waveguide woodwind instrument tonehole implementation which, in a single model, characterizes all states of the hole from open to closed. This efficient implementation produces results which agree well with previous acoustical analyses of the tonehole. A similar model is also presented for the register hole. A complete woodwind instrument model with many toneholes and register hole(s) is implemented in a cross-platform, C++ real-time computer programming environment. A new wind controller created to control the woodwind model is also briefly discussed.

INTRODUCTION

Computer modeling of musical instruments has been an active area of research for nearly two decades. Because of inherent nonlinearities in such systems, time-domain models have been of particular interest. Most time-domain modeling techniques for complete woodwind instrument systems involve the convolution of a pre-calculated or measured instrument reflection function with a nonlinear driving mechanism (1). Digital waveguide (DW) modeling (2) is a technique which simulates traveling-wave propagation along the length of a woodwind instrument bore using digital delay lines. Thus, a distributed model of the air column is used to continuously calculate the instrument reflection function, which allows variation of the resonator parameters in a physical manner during a simulation.

Most previously reported models of a woodwind tonehole have characterized only one state of the hole (open or closed). A dynamic DW tonehole model was presented (3), but this neglected the effects of closed holes.

TONEHOLE MODELS

A method was previously reported (4) for converting the continuous-time woodwind tonehole model of Keefe (5) to a discrete-time two-port scattering junction model for implementation in the digital waveguide (DW) domain. The results using this technique are shown in Figure 1 and compared with reproduced results using the technique of Keefe (6) for a simple flute air column with six toneholes. Discrepancies between the two methods are mainly evident in early closed hole reflections. Keefe's results were calculated for a frequency range of 10 kHz and subsequently smoothed in the time-domain with a hamming window (7). By trial and error, a lowpass filter with a cutoff frequency around 4 kHz was found to best reproduce Keefe's results. The DW results were obtained at a sampling rate of 44.1 kHz and then lowpass filtered to a 10 kHz bandwidth to correspond with the calculations of (6). Further lowpass filtering is inherent from the Lagrangian, delay-line length interpolation technique used in this model (8). Because such filtering is applied at different locations along the air column and is dependent on the particular fractional delay length modeled, a cumulative effect is difficult to accurately determine. As diagrammed in Figure 2, that portion of the signal reflected at the first tonehole is affected by only two interpolation filters, that at the second tonehole reflection is affected by four filtering operations, etc. Thus, *early* reflections in the DW model results are less lowpass filtered than the results of (6). It should be noted that each fractional delay interpolation filter in this implementation can be combined with a lossy propagation filter, which models lumped thermoviscous losses along its corresponding segment of the air column and which is also given by a lowpass frequency response. In this way, the inaccuracies inherent in low-order delay length interpolation filters can often be minimized. Alternately, higher-order interpolation filters can be used which introduce minimal frequency magnitude distortion.

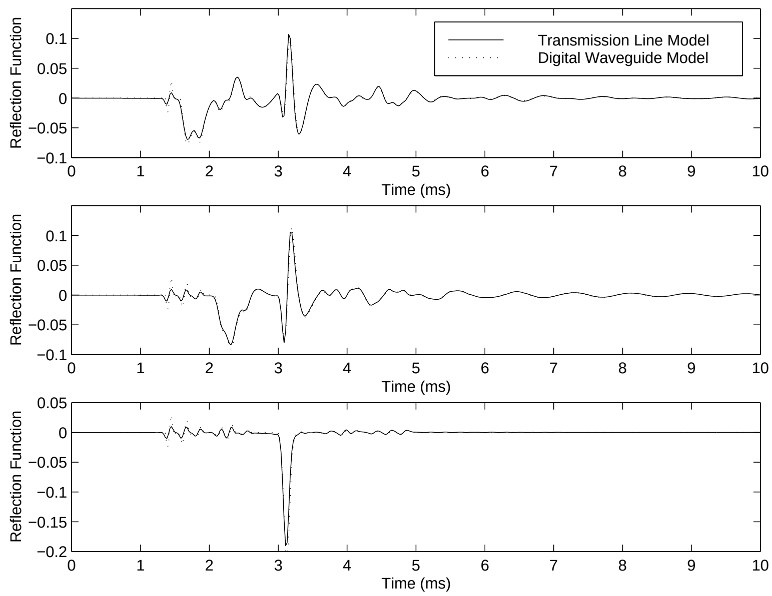

FIGURE 1. Calculated reflection functions for a simple flute air column [see (6)]. Transmission line model vs. DW two-port model with one hole closed (top), three holes closed (middle), and six holes closed (bottom).

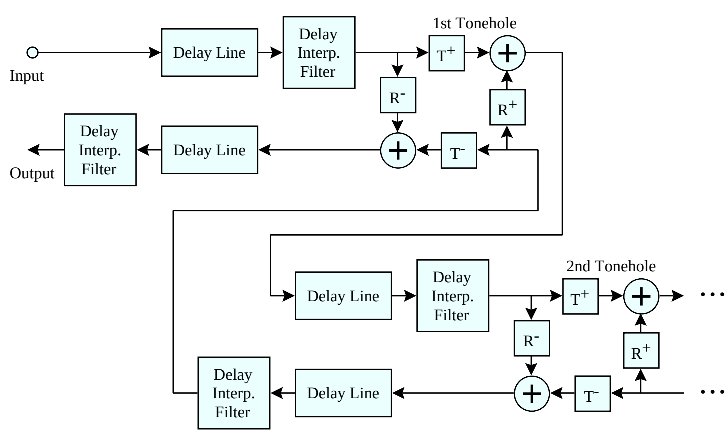

FIGURE 2. Digital waveguide two-port tonehole implementation scheme, including delay-line length interpolation filters.

For the purpose of real-time modeling, the two-port implementation has a particular disadvantage: the two lumped characterizations of the tonehole as either closed or open cannot be efficiently unified into a single tonehole model. While it is possible to develop a cross-fading/interpolation scheme to simulate "half-holing", this would require that two simultaneous models be run to simulate just one tonehole. It is preferable to have one model with adjustable parameters to simulate the various states of the tonehole, from closed to open and all states in between.

To this end, it is best to consider a distributed model of the tonehole, such that "fixed" portions of the tonehole structure are separated from the "variable" component. The junction of the tonehole branch with the main air column of the instrument can be modeled in the DW domain using a three-port scattering junction, as described in (4). This method inherently models only the shunt impedance term of the Keefe tonehole characterization, however, the negative length correction terms implied by the series impedances can be approximated by adjusting the delay line lengths on either side of the three-port scattering junction. The other "fixed" portion of the tonehole is the short branch segment itself, which is modeled in the DW domain by appropriately sized delay lines. This leaves only the characterization of the open/closed tonehole end. A simple inertance model of the open hole end offers the most computationally efficient solution. The impedance of the open end is then given by

$$Z_e^{(o)}(s) = \frac{\rho t}{S_e} s, \tag{1}$$

where ρ is the density of air, S_e is the cross-sectional area of the end hole, t is the effective length of the opening ($\approx S_e^{1/2}$), and s is the Laplace transform frequency variable. The open-end reflectance is

$$\mathcal{R}_e^{(o)}(s) \triangleq \frac{P_e^-(s)}{P_e^+(s)} = \frac{Z_e^{(o)}(s) - Z_{0b}}{Z_e^{(o)}(s) + Z_{0b}} = \frac{ts - c}{ts + c}, \tag{2}$$

where Z_{0b} is the characteristic impedance of the tonehole branch waveguide and c is the speed of sound. An appropriate discrete-time filter implementation for $\mathcal{R}_e^{(o)}$ can be obtained using the conformal bilinear transform from the s-plane to the z-plane (9, pp. 415-430), with the result

$$\mathcal{R}_e^{(o)}(z) = \frac{a - z^{-1}}{1 - az^{-1}}, \tag{3}$$

where

$$a = \frac{t\alpha - c}{t\alpha + c} \tag{4}$$

and α is the bilinear transform constant which controls frequency warping. A good low-frequency discrete-time fit is achieved for $\alpha = 2f_s$. The discrete-time reflectance $\mathcal{R}_e^{(o)}(z)$ is a first-order allpass filter, which is consistent with reflection from a "masslike" impedance. It is possible to simulate the closing of the tonehole end by taking the end hole radius (or S_e) smoothly to zero. In the above implementation, this is accomplished simply by varying the allpass coefficient between its fully open value and a value nearly equal to one. With $a \approx 1$, the reflectance phase delay is nearly zero for all frequencies, which corresponds well to pressure reflection at a rigid termination. A complete implementation scheme is diagrammed in Figure 3. Figure 4 shows the reflection functions obtained using this model in comparison to the Keefe transmission-line results. This efficient model of the tonehole produces results very much in accord with the more rigorous model. A more accurate model of the tonehole branch end, which is not pursued here, would include a frequency-dependent resistance term and require the variation of three first-order filter coefficients.

REGISTER HOLE MODELS

Woodwind register holes are designed to discourage oscillations based on the fundamental air column mode and thus to indirectly force a vibratory regime based on higher, more stable resonance

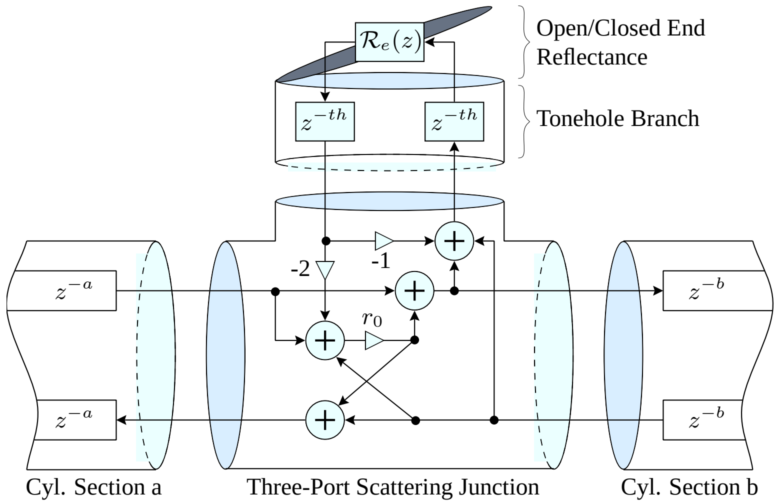

FIGURE 3. "Distributed" digital waveguide tonehole implementation.

frequencies. A register vent functions both as an acoustic inertance and an acoustic resistance (10). It is ideally placed about one-third of the distance from the excitation mechanism of a cylindrical-bored instrument to its first open hole. Sound radiation from a register hole is negligible.

The DW implementation of a register hole can proceed in a manner similar to that for the tonehole. The series impedance terms associated with toneholes are insignificant for register holes and can be neglected. Modeling the open register hole as an acoustic inertance in series with a constant resistance, its input impedance as seen from the main bore is given by

$$Z_{rh}^{(o)}(s) = \frac{\rho t}{S_{rh}} s + \xi, \tag{5}$$

where ρ is the density of air, t is the effective height, S_{rh} is the cross-sectional area of the hole, ξ is the acoustic resistance, and s is the Laplace transform frequency variable. Proceeding with a two-port DW implementation, the register hole is represented in matrix form by

$$\begin{bmatrix} P_1^- \\ P_2^+ \end{bmatrix} = \begin{bmatrix} \mathcal{R}^- & \mathcal{T}^- \\ \mathcal{T}^+ & \mathcal{R}^+ \end{bmatrix} \begin{bmatrix} P_1^+ \\ P_2^- \end{bmatrix} = \frac{1}{Z_0 + 2Z_s} \begin{bmatrix} -Z_0 & 2Z_s \\ 2Z_s & -Z_0 \end{bmatrix} \begin{bmatrix} P_1^+ \\ P_2^- \end{bmatrix}, \tag{6}$$

where the open register hole shunt impedance is given by $Z_{rh}^{(o)}$ and Z_0 is the characteristic impedance of the main air column. The reflectances and transmittances are equivalent at this junction for wave components traveling to the right or to the left. As $\mathcal{T} = 1 + \mathcal{R}$, a one-filter form of the junction is possible. Using the bilinear transform, an appropriate discrete-time implementation for $\mathcal{R}_{rh}$ is given by

$$\mathcal{R}_{rh}^-(z) = \mathcal{R}_{rh}^+(z) = \frac{-c\left(1 + z^{-1}\right)}{(\zeta + \alpha\psi) + (\zeta - \alpha\psi)\, z^{-1}}, \tag{7}$$

where

$$\zeta = c + 2S_0\xi/\rho \qquad \text{and} \qquad \psi = 2S_0 t/S_{rh}, \tag{8}$$

S_0 is the cross-sectional area of the main air column, and α is the bilinear transform constant which controls frequency warping. Once again, a good low-frequency discrete-time fit is achieved for $\alpha = 2f_s$. Assuming the closed register hole has neglible effect in the acoustic model, simulated closure of the register hole in this implementation is achieved by ramping the reflectance filter gain

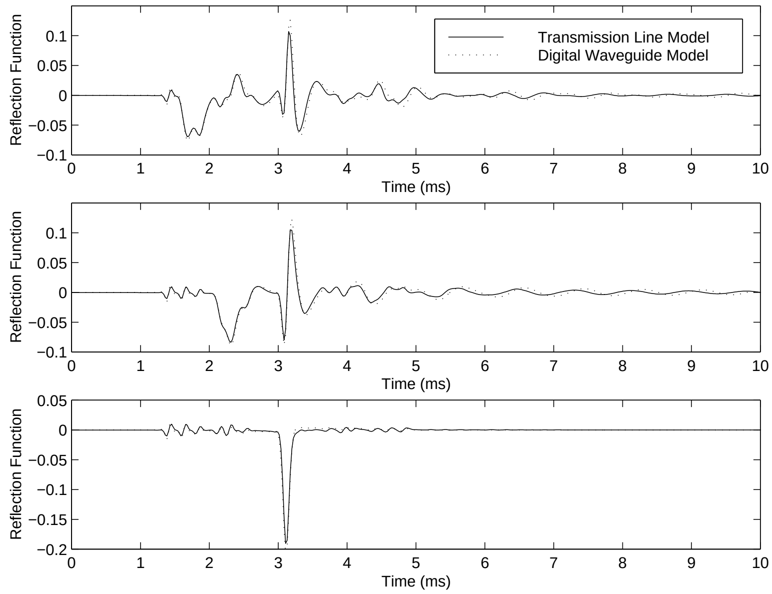

FIGURE 4. Calculated reflection functions for a simple flute air column [see (6)]. Transmission line model vs. DW "distributed" tonehole model with one hole closed (top), three holes closed (middle), and six holes closed (bottom).

to zero. This implementation is similar to that of (3), though resistance effects were not accounted for in that study.

As discussed by Benade (10, p. 459), a misplaced register hole will raise the frequency of the second air column mode by an amount proportional to its displacement from the ideal location (in either direction). Such behavior is well demonstrated when this register hole implementation is added to the real-time clarinet model. The instrument builder and computer programmer are thus faced with the same dilemma: how many register vents to create and where best to put them!

THE PROGRAMMING ENVIRONMENT

Real-time implementations of the models discussed in this paper were carried out using a platform-independent, floating-point, C++ environment created by Perry R. Cook called *Synthesis ToolKit (STK)* (11). A significant number of signal processing unit generator objects are provided with the toolkit, as well as tools for I/O streaming and file generation. The newest release of *STK* has been ported to NeXTStep, Irix, and Linux flavors of Unix, and a separate Windows95 version exists as well. In addition to real-time output, *STK* supports simultaneous creation of NeXT/SGI (.snd) soundfiles, and/or Win (.wav) soundfiles, and/or Matlab (.mat) matfiles.

Parameter control in *STK* is handled by a text-based protocol called SKINI. SKINI is user-extensible and is fully compatible with MIDI. For the latest information regarding *STK* and SKINI, consult `http://www.cs.princeton.edu/~prc/` and `http://www-ccrma.stanford.edu/~gary/`.

REALTIME MODEL CONTROL

Simple computer graphical user interfaces have been created to control the tonehole and register hole parameters in the *STK* real-time implementation. Unfortunately, existing MIDI controllers do not provide the necessary level of control. MIDI wind controllers, for example, produce only `NoteOn` and `NoteOff` messages when a key is depressed, providing no intermediate key position information. Further, any attempt to use unconventional fingerings on such controllers are impossible. Therefore, it was necessary to construct a new controller which would provide "continuous" and independent key position information.

The *HolyController* prototype was created by "retrofitting" an existing Yamaha WX11 wind controller. That is, the WX11 was used for breath pressure and velocity control and its key mechanism was reworked to provide a separate stream of MIDI information regarding each key position. Force sensing resistors (FSR™) by Interlink Electronics were positioned under the WX11 keys and connected to a BASIC Stamp II (BS2-IC) microprocessing unit. The MIDI stream emitted by the BS2-IC is merged with that from the WX11 using a generic MIDI merging box, and then input to the *STK* model. Each tonehole and register hole model parameter is controlled with a distinct MIDI `ControlChange` number and value, such that 128 states are possible between fully closed and fully open.

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